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ROBUST PROCEDURES FOR MULTI-SAMPLE LOCATION PROBLEMS.

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by

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ABSTRACT

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Robust procedures are proposed for testing the equality of several group means for equal and unequal group variances. These statistics are obtained by modifying the usual F test for equal variances and the Welch's W and Brown-Forsythe's F^* for unequal variances by using the trimmed mean and the sine-wave M-estimator. Approximate distributions of these new statistics are obtained under normality. Their performances are evaluated by Monte Carlo sampling experiments under various long-tailed symmetric distributions, comparing among themselves and with some nonparametric tests. For the cases studied, the proposed statistics are shown to be robust in the sense that they are mildly suboptimal under exact normality but perform well over a broad spectrum of long-tailed symmetric distributions.

To my parents

and

to my wife

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INTRODUCTION

1.1 THE CONCEPT OF ROBUSTNESS

The word "robust" is interpreted in many different meanings in statistics. A common interpretation is insensitivity of a statistical procedure against some departures of assumptions on which the procedure is based on. A narrow view of robustness is mainly concerned with whether the probability statement associated with a statistical procedure is valid in spite of some violations of underlying assumptions - robustness of validity. A broader view includes not only the robustness of validity but also the robustness of efficiency - how efficient a procedure is to extract information concerning the underlying population from a sample in the presence of some departures of assumptions. We will use the word "robust" here in the broader sense.

Many statistical procedures are based on the normal law and generally they are optimal under normality. However, it has been realized that in real life, long-tailed distributions occur frequently and procedures optimal under normality usually perform poorly under long-tailed distributions. On the other hand, their performance under short-tailed distributions is rather good. Hence, we focus our attention on long-tailed distributions.

The basic principle in developing a robust procedure is that we look for a procedure which may be mildly suboptimal at an ideal distribution (normal in many cases) but performs well over a broad class of distributions.

1.2 MULTI-SAMPLE LOCATION PROBLEMS

Consider a multi-sample location problem of testing the equality of several group means. Let X_{ij} be the j-th observation, $j = 1, 2, \ldots, n_i$, from i-th group and let $E(X_{ij}) = \mu_i$, $V(X_{ij}) = \sigma_i^2$, $i = 1, 2, \ldots, c$, be unknown. In order to test the null hypothesis of equal group means, the usual F statistics

$$F = \frac{\sum_{i=1}^{c} n_i(\bar{X}_i - \bar{X})^2/(c-1)}{\sum_{i=1}^{c}\sum_{j=1}^{n_i}(X_{ij} - \bar{X}_i)^2/(N-c)}$$

is commonly used assuming independence of observations, normality of parent distributions and equality of group variances. Under these assumptions, F is optimal - F is UMP in a properly restricted class of tests [23].

Now we ask if F is robust against some departures from these assumptions. The followings are known:

1. It is robust especially in validity against mild nonnormality [6, 20, 23].
2. It is conservative and inefficient under long-tailed distributions [6, 20] and the conservatism and inefficiency increase as the degree of long-tailedness increases.

3. The effect of kurtosis on F is more serious than that of skewness [8, 20, 26].
4. It is very sensitive against heteroscedasticity [4, 12].

The existence of correlation between the numerator and denominator in the F-ratio under nonnormality contributes largely to the robustness of F, particularly in validity against mild nonnormality for equal variances case [6, 20]. However, as mentioned in Point 2 above, F is conservative and inefficient under long-tailed situations.

For the sensitivity of F to heteroscedasticity, Welch [28] proposed a less sensitive statistics for the two-sample case, which was extended to multi-sample by James [14] and Welch himself [29]. Their statistics have the same numerator but different denominator and they use different approximations to obtain critical values. Brown and Forsythe [4] proposed another one by modifying F. In their paper they also investigated the performance of F, James' J, Welch's W and their own F* under various combinations of n_i's, μ_i's and σ_i^2's for normality. Their findings are:

1. These statistics except F keep significance level within a reasonable range for not too small sample sizes.
2. Welch's approximation is better than James'.
3. When variances are equal, both W and F* are lower but close to F in power.
4. When extreme means have small variances, W is more powerful than F* and vice versa for the opposite situation.

These statistics give us protection against heterogeneity of variances under normality but not necessarily under long-tailedness.

Yuen [32] showed that the two-sample Welch's statistic is conservative and inefficient under long-tailed underlying distributions and proposed a robust statistic against both heterogeneity and long-tailedness.

Obviously the performance of these statistics and F depends on the location estimator (the sample mean) and its matching variance estimator (a constant times the sample variance) used in the statistics. Since the sample mean performs poorly under long-tailed distributions, we think that we may obtain robust statistics against long-tailedness if we replace the sample mean and the sample variance in these statistics by a robust location estimator and its matching variance estimator. Hence, we first seek good robust location estimators with known matching variance estimators.

Following the Tukey and McLaughlin's view [27] that an adequate mastery of the symmetrical distribution case is a natural first step in our progress from statistical techniques based on normality, and because kurtosis has larger effect on F than skewness and a skewness problem can somewhat be solved by using transformation, we will concentrate our attention on symmetric long-tailed distributions in this study.

1.3 ROBUST LOCATION ESTIMATORS

The majority of location estimators can be classified into three classes [13]: namely, the M-estimator, the L-estimator, and the R-estimator.

1. M-estimator (maximum likelihood type estimator)

   Let T be an estimator for the location parameter μ which minimizes

$$\sum_{j=1}^{n} \rho(X_j - T) \quad \text{for some function } \rho,$$

or be a solution of $\sum_j \psi(X_j - T) = 0$ where $\psi = \partial\rho/\partial\mu$. The choice of ρ or ψ determines T. If $\rho = -\log[f(x - \mu)]$ where f is the density of the underlying distribution, then T becomes the ordinary maximum likelihood estimator.

Example: Andrews' sine-wave M-estimator is defined by

$$\psi(x) = \begin{cases} \text{sine}(x/k) & \text{for } |x| < k\pi \\ 0 & \text{otherwise} \end{cases}$$

for some positive constant k.

2. L-estimator (linear combination of order statistics)

For a signed measure M on (0, 1), the functional $T(F) = \int F^{-1}(t)M(dt)$ defined on distribution functions induces an estimator

$$T = T(F_n) = \sum_{j=1}^{n} a_{nj} X_{(j)}$$

where F_n is an empirical distribution function, $a_{nj} = M\{((j-1)/n, j/n]\}$ and $X_{(j)}$ is the j-th order statistic.

Example: The g-trimmed mean $(0 \le g \le 0.5)$ is defined by $T = \sum_j a_{nj} X_{(j)}$ with $a_{nj} = 1/(n - 2[gn])$ for $[gn] + 1 \le j \le n - [gn]$, 0 otherwise, and [x] is the greatest integer less than or equal to x.

3. R-estimator (nonparametric statistic)

This estimator is induced from a linear rank test.

Example: Hodges-Lehmann estimator which is defined by $T = \text{median}\{(X_i + X_j)/2\}$ and induced from the Wilcoxon rank test.

Hampel [10] defined the influence curve by

$$IC(x:F,\ T) = \lim_{t \to 0} \{T[(1 - t)F + t\delta_x] - T(F)\}/t$$

which gives the suitably scaled differential influence of one additional observation with value x when $n \longrightarrow \infty$, where δ_x is the distribution function of point mass 1 at x. Figure 1 shows the influence curves of estimators given in the examples above and the sample mean for a continuous distribution with unimodal density symmetric about 0 [10,12].

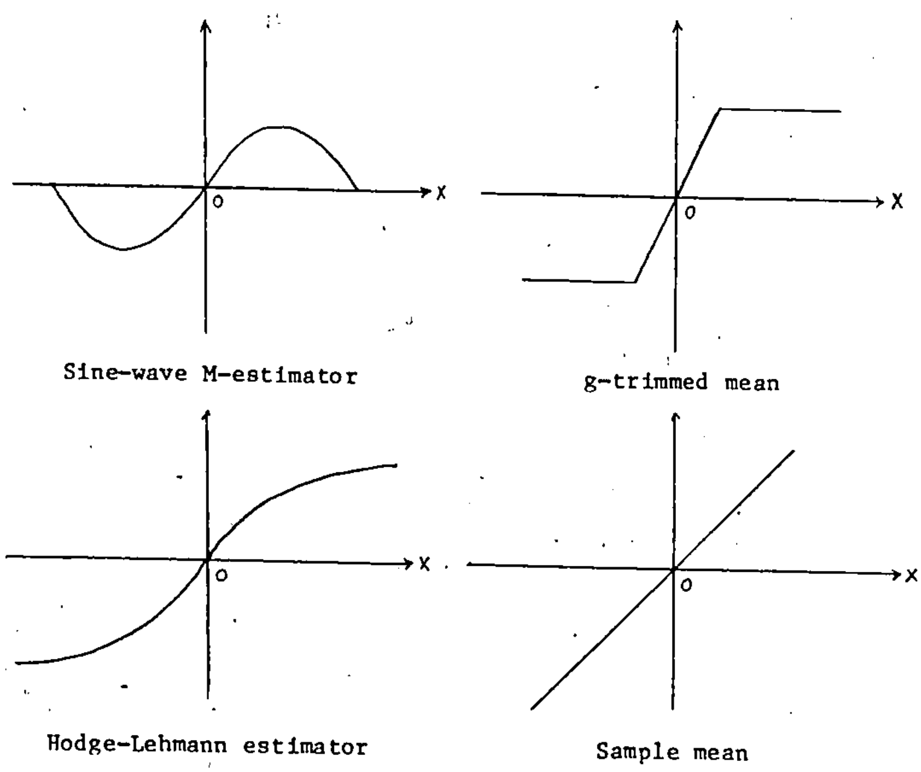


Figure 1: Influence curves of some location estimators

Boundedness of IC is desirable bacause extreme values will have bounded influence on the estimator. The sensitivity of the sample mean to extreme values is well illustrated by its IC.

In the Princeton study [1], the performance and properties of some 68 location estimators were assessed under 40 different sampling situations. It was shown in the study that the trimmed mean and the sine-wave M-estimator perform fairly well over a wide spectrum of underlying distributions. We will use these two robust estimators to modify F, W and F* to obtain robust statistics against long-tailedness.

1.4 DIRECTION AND SCOPE OF THE STUDY

In Chapter 2, the trimmed F for equal variances and the trimmed W and F* statistics for unequal variances are proposed. The asymptotic distributions of trimmed F and W are derived and the approximate distributions of all three statistics are investigated by Monte Carlo techniques.

In Chapter 3, the sine-wave F, W and F* statistics for equal and unequal variances are proposed and their approximate behaviour is studied by Monte Carlo methods.

In Chapter 4, the proposed statistics in Chapters 2 and 3 are evaluated by comparisons among themselves along with some nonparametric test procedures by Monte Carlo sampling experiments. Recommendations are given based on the results of this evaluation.

In Chapter 5, a general discussion of the results presented in this dissertation is given and suggestions for further study are made.

Chapter II

TRIMMED STATISTICS FOR MULTI-SAMPLE LOCATION PROBLEMS

2.1 THE TRIMMED MEAN AND WINSORIZED SUM OF SQUARED DEVIATIONS

Let $X_1, X_2, \ldots, X_n$ be an ordered sample of size n and let $k = [gn] + 1$ where $[x]$ is the largest integer $\leq x$. For $0 < g \leq 0.25$, the g-trimmed mean and the g-Winsorized mean of the sample are defined, respectively, as

$$\bar{X}_{tg} = [\sum_{j=k+1}^{n-k} X_j + (1-\epsilon)(X_k + X_{n-k+1})]/(n - 2gn),$$

$$\bar{X}_{wg} = \{\sum_{j=k+1}^{n-k} X_j + k[(1-\epsilon)(X_k + X_{n-k+1}) + \epsilon(X_{k+1} + X_{n-k})]\}/n$$

where $\epsilon = gn - [gn]$.

The g-Winsorized sum of squared deviations is then

$$SSD_{wg} = \sum_{j=k+1}^{n-k} (X_j - \bar{X}_{wg})^2 + k\{[(1-\epsilon)X_k + \epsilon X_{k+1} - \bar{X}_{wg}]^2$$
$$+ [(1-\epsilon)X_{n-k+1} + \epsilon X_{n-k} - \bar{X}_{wg}]^2\}.$$

This fractional trimming/Winsorization definition is somewhat different from the usual integral trimming/Winsorization definition in that not only integer number of observations are deleted or replaced but a fraction (ϵ) is also taken into account. Lim and Fung [17] used this fractional trimming/Winsorization in defining their sequential trimmed t in order to have the same trimming/Winsorization proportion (g) at each stage of sampling. In multi-sample problems where sample sizes may be different, we also need this fractional definition so that we can apply the same proportion to each sample.

2.2 DEVELOPMENT OF TRIMMED STATISTICS IN TESTING OF LOCATION

In 1963, Tukey and McLaughlin [27] proposed the one-sample trimmed t formed by the ratio of the trimmed mean to the square root of the Winsorized sum of squared deviations, that is,

$$t_{tg} = \frac{\bar{X}_{tg} - \mu}{\sqrt{SSD_{wg}/[h(h-1)]}}$$

where $h = n - 2[gn]$. They suggested using this statistic for hypothesis testing and confidence interval by referring it to the Student's t table with $h - 1$ degrees of freedom. In the same year, Wonnacott [30] studied the power of the one-sample trimmed t by comparing with Student's t and Walsh's nonparametric procedure under normal and symmetric Johnson's distributions. His general conclusion is that the performance of the trimmed t is very good.

In 1973, Yuen and Dixon [33] extended the one-sample trimmed t to the two-sample case for equal variances. The two-sample trimmed t is defined by

$$t_{t/w} = \frac{(\bar{X}_{1tg} - \bar{X}_{2tg}) - (\mu_1 - \mu_2)}{\sqrt{(SSD_{1wg} + SSD_{2wg})(1/h_1 + 1/h_2)/(h_1 + h_2 - 2)}}$$

where $h_i = n_i - 2[gn_i]$, $i = 1, 2$, and it is suggested to approximate this statistic by the Student's t distribution with $h_1 + h_2 - 2$ degrees of freedom. They also reported good performance of $t_{t/w}$. There is a little loss of efficiency under normality but appreciable gain in power under mixtures of normals.

For unequal variances, in 1974, Yuen [32] modified the two-sample Welch's statistic to obtain a robust procedure against both heteroscedasticity and long-tailedness, and defined

$$t = \frac{(\bar{X}_{1tg} - \bar{X}_{2tg}) - (\mu_1 - \mu_2)}{\sqrt{(S^2_{1wg}/h_1 + S^2_{2wg}/h_2)}}$$

where $S^2_{iwg} = SSD_{iwg}/(h_i - 1)$, $i = 1, 2$.

Approximate critical values can be obtained from the Student's t table with f degrees of freedom where

$$1/f = a^2/(h_1 - 1) + (1 - a)^2/(h_2 - 1)$$

and

$$a = (S^2_{1wg}/h_1)/(S^2_{1wg}/h_1 + S^2_{2wg}/h_2).$$

Note that integral trimming/Winsorization was used in the one-sample and two-sample trimmed t.

In 1980, Lim and Fung [17] used the trimmed mean in defining their one-sample sequential trimmed t statistic which is robust under long-tailed situations.

Besides good performance under long-tailed distributions, the trimmed statistics also possess other advantages such as simplicity in both concept and application, and known moderate sample approximate behaviour.

2.3 TRIMMED F FOR EQUAL VARIANCES

2.3.1 Formulation of the Trimmed F

We define the g-trimmed F by

$$F_t(g) = \frac{\sum_{i=1}^{c} h_i (\bar{X}_{itg} - \bar{X}_{tg})^2/(c-1)}{\sum_{i=1}^{c} SSD_{iwg}/(H-c)}$$

where $h_i = n_i(1-2g)$,

$H = \sum_i h_i = N(1-2g)$, $(N = \sum_i n_i)$

$\bar{X}_{itg}$ = the g-trimmed mean of the i-th group,

$\bar{X}_{tg} = \sum_i h_i \bar{X}_{itg}/H$

SSD_{iwg} = the g-Winsorized sum of squared deviations of the i-th group.

Note that when $g = 0$, $F_t(g) = F$.

In order to use the trimmed statistic for testing, we need critical values. The normal distribution still serves as our ideal one and hence, we want to obtain the distribution of $F_t(g)$ under normality. The exact distribution is very difficult if not impossible to obtain analytically. However, Monte Carlo techniques can be used to obtain good critical values by working with the empirical distribution function of $F_t(g)$. This method is not easy either because it requires a tremendous amount of calculations. It will be shown in Section 2.5 that $F_t(g)$ is asymptotically distributed as chi-square with c − 1 degrees of freedom. However, the convergence is not fast enough to use the asymptotic theory for moderate sample sizes. Fortunately, past experience guided us to conjecture that $F_t(g)$ will follow approximately an F distribution with (c − 1, H − c) degrees of freedom for not too small sample sizes. This conjecture will be verified by Monte Carlo sampling experiments in the next subsection.

2.3.2 Approximate Behaviour of the Trimmed F under Normality

A set of $N = \sum_i n_i$ observations from a standard normal distribution were generated using the IMSL normal random number generator GGNPM and assigned to consecutive groups (c = 3, 4, 6). A different seed for generating random numbers was used for each set. The $F_t(g)$ statistic was calculated and a check was made to see if the null hypothesis was accepted or rejected at $\alpha = 1$, 5, 10% by comparing $F_t(g)$ with the critical values of the F table with (c − 1, H − c) degrees of freedom. The above experiment was repeated 5,000 times and the proportion of times the null hypothesis was rejected was obtained for different levels of trimming.

Table 1 shows the results. We see that the tail probabilities of $F_t(g)$ are slightly higher than that of the F we conjectured especially when g is high and n_i's are small. However, when the sample in each group is of size 10 or more, the approximation seems satisfactory.

TABLE 1

Empirical significance levels of $F_t(g)$ under normality

Sample sizes (n_i)	Nominal α(%)	% of trimming (100g%) 0	5	10	15	20	25
5,5,5	1	0.9	0.8	0.9	1.0	1.3	0.8
	5	4.4	4.4	4.4	4.9	5.7	5.0
	10	10.0	9.9	9.6	10.4	10.8	10.2
7,7,7	1	1.0	1.0	1.1	0.9	1.0	1.4
	5	5.2	4.8	5.1	5.2	5.2	6.1
	10	10.5	10.5	10.6	10.3	10.7	11.6
10,10,10	1	1.2	1.2	1.0	1.1	1.4	1.6
	5	4.8	4.9	4.8	4.9	5.5	5.4
	10	9.7	9.5	9.6	9.7	9.7	10.2
20,20,20	1	1.1	1.1	1.2	1.2	1.5	1.5
	5	4.7	4.7	5.0	5.2	5.2	5.5
	10	9.4	9.4	9.5	9.5	9.9	10.5
5,10,15	1	0.8	0.8	0.7	0.9	1.1	1.1
	5	4.8	4.8	5.0	5.2	5.5	6.0
	10	9.5	9.7	9.8	10.0	10.3	10.3
7,7,7,7	1	1.2	1.1	1.3	1.3	1.2	1.6
	5	5.1	5.3	5.3	5.5	5.7	6.2
	10	9.8	9.9	10.2	10.6	10.7	11.9
10,10,10,10	1	1.1	1.1	1.1	1.0	1.1	1.0
	5	4.9	5.1	5.2	5.2	5.6	5.7
	10	10.3	10.2	9.9	10.3	10.5	11.1
10,15,15,20	1	0.8	0.9	1.0	1.0	1.2	1.1
	5	4.7	4.5	4.7	4.9	5.3	5.6
	10	9.7	9.4	9.6	9.9	9.8	10.7
20,20,20,20	1	0.9	0.8	1.0	1.1	1.2	1.1
	5	5.0	4.9	4.7	4.8	5.4	5.6
	10	10.1	10.0	10.1	10.1	10.3	10.6
5,10,15,20	1	0.9	0.9	0.9	1.2	1.0	1.1
	5	4.7	4.5	4.8	5.0	5.1	5.2
	10	9.8	9.5	9.4	9.5	9.9	10.3
5,5,20,20	1	0.8	1.0	1.0	1.0	1.3	1.2
	5	4.6	4.6	4.4	4.9	5.4	5.7
	10	9.4	9.6	9.4	9.7	10.4	10.5
7,7,7,7,7,7	1	0.9	0.9	1.2	1.3	1.3	1.6
	5	4.7	4.9	5.0	4.9	5.3	6.3
	10	9.7	9.9	10.5	10.6	10.8	12.1
10,10,10,10,10,10	1	0.8	0.9	1.0	1.0	1.2	1.1
	5	4.9	4.9	4.9	5.0	4.7	5.3
	10	9.5	9.8	9.5	9.9	10.2	10.5
20,20,20,20,20,20	1	0.8	0.9	0.9	0.9	0.8	0.8
	5	4.9	5.0	4.4	4.8	5.0	4.9
	10	9.7	9.7	9.9	10.2	10.1	10.2
5,10,10,15,15,20	1	0.9	0.9	1.0	0.9	0.9	1.1
	5	5.0	5.0	4.8	4.7	4.9	5.0
	10	10.3	10.0	9.9	10.0	10.6	10.7

2.4 TRIMMED STATISTICS FOR UNEQUAL VARIANCES

2.4.1 The Trimmed W

As mentioned in Section 1.2, Welch [29] proposed a test statistic for unequal variances, namely,

$$W = \frac{\sum_{i=1}^{c} w_i(\bar{X}_i - \tilde{X})^2/(c-1)}{1 + [2(c-2)/(c^2-1)]\sum_{i=1}^{c}(1 - w_i/w)^2/(n_i - 1)}$$

where $w_i = n_i/S_i^2$ (S_i^2 is the sample variance of i-th group),

$w = \sum_i w_i$,

$\bar{X}_i$ = i-th group sample mean,

$\tilde{X} = \sum_i w_i \bar{X} / w$.

When the group means are equal and the parent distributions are normal, W is approximately distributed as an F with $(c - 1, f)$ degrees of freedom where f is implicitly defined by

$$1/f = [3/(c^2 - 1)] \sum_{i=1}^{c} (1 - w_i/w)^2/(n_i - 1).$$

The modification of W using the trimmed mean and Winsorized sum of squared deviations is

$$W_t(g) = \frac{\sum_{i=1}^{c} w_i(\bar{X}_{itg} - \tilde{X}_{tg})^2/(c-1)}{1 + [2(c-2)/(c^2-1)]\sum_{i=1}^{c}(1 - w_i/w)^2/(n_i - 1)}$$

where $w_i = h_i/S_{iwg}^2$ $(S_{iwg}^2 = SSD_{iwg}/(h_i - 1))$,

$$w = \sum_i w_i ,$$

$$\tilde{X}_{tg} = \sum_i w_i \bar{X}_{itg}/w.$$

When $g = 0$, $W_t(g)$ reduces to W.

From the approximate behaviour of other trimmed statistics, we conjectured that under H_o and normality, $W_t(g)$ is approximately distributed as an F distribution with $(c - 1, \ f_t)$ degrees of freedom where

$$1/f_t = [3/(c^2 - 1)] \sum_{i=1}^{c} (1 - w_i/w)^2 /(h_i - 1).$$

The verification of this conjecture by sampling experiments will be shown in Subsection 2.4.3.

2.4.2 The Trimmed F*

The Brown and Forsythe's modified F for unequal variances [4] is

$$F^* = \frac{\sum_{i=1}^{c} n_i(\bar{X}_i - \bar{X})^2}{\sum_{i=1}^{c}(1 - n_i/N)S_i^2}.$$

Approximate critical values of this statistic are obtained from the F table with $(c - 1, \ f^*)$ degrees of freedom where f^* is implicitly defined by

$$1/f^* = \sum_i v_i^2/(n_i - 1),$$

and $v_i = (1 - n_i/N)S_i^2/[\sum_j (1 - n_j/N)S_j^2]$.

The g-trimmed F* is, then

$$F_t^*(g) = \frac{\sum_{i=1}^{c} h_i(\bar{X}_{itg} - \bar{X}_{tg})^2}{\sum_{i=1}^{c}(1 - h_i/H)S_{iwg}^2}.$$

Our conjecture is again that under H_0 and normality $F_t^*(g)$ will follow approximately an F distribution with $(c - 1, f_t^*)$ degrees of freedom where

$$1/f_t^* = \sum_i v_i^2/(h_i - 1),$$

and $v_i = (1 - h_i/H)S_{iwg}^2/[\sum_j (1 - h_j/H)S_{jwg}^2]$.

As before, $F_t^*(g) = F^*$ when $g = 0$. Note that when $c = 2$, both $W_t(g)$ and $F_t^*(g)$ become the square of the two-sample trimmed t for unequal variances [32].

2.4.3 Approximate Behaviour of $W_t(g)$ and $F_t^*(g)$ under Normality

Monte Carlo techniques were again employed to verify the approximate behaviour of $W_t(g)$ and $F_t^*(g)$. Four groups and six groups with equal and unequal sample sizes were tried under a variety of σ_i's, The experiment is similar to that of trimmed F. Five thousand samples were replicated, and the same samples were used for both $W_t(g)$ and $F_t^*(g)$.

Table 2 shows the results of the Monte Carlo study. The first line of the results is the empirical α's of $W_t(g)$ and the second line is those of $F_t^*(g)$. As can be seen from the table, $W_t(g)$ tends to be a little liberal as g increases. However, if we restrict g to [0, 0.15], the values of empirical α are in reasonable ranges. On the other hand, the level of $F_t^*(g)$ does not seem to be much affected by changing g.

The liberalism of $W_t(g)$ could be corrected by multiplying the statistic by an appropriate decreasing function of g, but it is desirable to keep the statistic as simple as possible. Avoiding any modification, we are content with the results of $W_t(g)$, restricting $0 \leq g \leq 0.15$. The approximate behaviour of $F_t^*(g)$ is also satisfactory in the sense that the behaviour of $F_t^*(g)$ is not any worse than F^*. Note that we have only investigated the cases which the ratio of minimum to maximum variance does not exceed 9.

TABLE 2

Empirical significance levels of $W_t(g)$ and $F_t^*(g)$ under normality

Sample sizes (n_i)	Standard deviations	Nominal α(%)	% of trimming (100g%) 0	5	10	15	20	25
10,10,10,10	1,1,1,1	1	1.0	1.0	1.1	1.1	1.2	0.8
			0.8	0.8	0.9	0.8	0.9	0.9
		5	4.8	5.0	5.4	5.3	5.8	5.4
			4.7	4.6	4.4	4.4	4.3	4.6
		10	9.7	9.7	10.5	10.5	11.2	10.5
			9.7	9.6	9.3	9.6	9.5	9.4
	1,2,2,3	1	1.0	0.9	1.1	1.0	1.7	1.3
			1.6	1.5	1.5	1.8	1.8	1.6
		5	5.7	5.6	6.0	6.1	6.9	6.5
			6.2	6.1	5.8	5.9	6.0	5.9
		10	10.9	10.8	11.1	11.1	12.7	12.0
			10.9	10.8	10.7	10.9	11.0	11.0
15,15,15,15	1,1,1,1	1	1.0	1.0	0.9	1.0	1.3	1.6
			0.8	0.8	0.7	0.8	0.8	1.0
		5	4.5	4.6	4.5	4.8	5.4	5.8
			4.9	4.8	4.6	4.6	4.5	4.8
		10	9.5	10.1	9.8	10.2	10.4	10.8
			9.5	9.6	9.8	9.6	9.2	9.4
	1,2,2,3	1	0.9	1.0	1.1	1.2	1.4	1.4
			1.3	1.5	1.5	1.4	1.6	1.7
		5	4.7	4.7	5.0	5.2	5.8	6.0
			6.0	5.7	5.5	5.6	5.5	5.2
		10	9.8	10.1	10.0	10.1	10.7	11.0
			10.8	10.6	10.4	10.2	10.3	10.3
20,20,20,20	1,1,1,1	1	0.9	0.8	0.8	0.9	1.0	1.2
			0.9	0.8	0.8	0.8	0.7	0.9
		5	4.9	5.0	5.1	5.3	5.2	5.5
			5.1	5.0	4.7	4.9	4.9	4.8
		10	9.6	9.7	10.1	10.2	10.6	11.0
			9.8	9.7	9.6	9.7	9.8	10.2
	1,2,2,3	1	1.1	1.1	1.3	1.4	1.5	1.7
			1.9	1.7	1.7	1.9	2.1	2.1
		5	5.0	4.7	5.1	5.3	5.7	6.2
			6.3	6.4	6.3	6.3	6.4	6.7
		10	10.2	10.2	10.4	10.7	11.2	11.6
			11.2	11.1	11.0	11.4	11.2	11.0
10,15,15,20	1,1,1,1	1	0.9	1.0	1.1	1.2	1.5	1.5
			1.0	1.0	1.1	1.0	0.8	0.8
		5	4.7	4.9	5.2	5.5	6.1	6.0
			4.8	4.9	4.9	5.0	5.0	5.0
		10	9.4	10.0	10.4	10.5	11.1	11.6
			9.2	9.6	9.7	9.7	9.7	10.0

TABLE 2 (continued)

Sample sizes (n_i)	Standard deviations	Nominal α(%)	% of trimming (100g%) 0	5	10	15	20	25
	1,2,2,3	1	0.9	1.0	0.9	1.1	1.2	1.5
			1.6	1.7	1.7	1.7	1.7	1.7
		5	5.0	5.1	5.0	5.1	5.7	5.9
			5.6	5.8	5.7	6.1	6.2	6.5
		10	9.9	10.0	9.9	10.1	11.0	11.4
			10.8	11.0	10.9	10.7	10.7	11.2
	3,2,2,1	1	1.1	1.1	1.2	1.3	1.8	1.7
			1.4	1.4	1.5	1.6	1.7	1.7
		5	5.0	5.2	5.5	5.5	6.3	6.3
			5.6	5.8	5.9	6.0	6.2	6.0
		10	10.0	10.1	10.3	10.5	12.0	12.2
			10.4	10.4	10.6	10.9	11.3	11.2
10,10,10, 10,10,10	1,1,2, 2,3,3	1	0.9	1.1	1.4	1.2	1.9	1.8
			1.8	1.6	1.6	1.3	1.4	1.3
		5	5.3	4.9	5.9	6.1	7.5	7.4
			6.1	5.8	5.7	5.8	5.9	5.6
		10	10.3	10.8	11.2	11.3	13.7	13.1
			11.1	10.9	10.0	10.4	10.5	10.5
20,20,20, 20,20,20	1,1,2, 2,3,3	1	1.0	1.3	1.1	1.2	1.6	1.9
			2.1	1.8	1.8	1.8	1.8	1.8
		5	5.4	5.3	5.6	6.2	6.4	6.8
			7.1	6.9	6.8	6.8	7.1	6.8
		10	10.6	10.7	10.8	11.6	11.5	12.6
			11.9	12.0	12.1	12.1	11.8	11.7
10,10,15, 15,20,20	1,1,1, 1,1,1	1	1.0	1.1	1.3	1.5	2.1	2.1
			1.2	1.3	1.2	1.1	1.0	1.0
		5	5.3	5.1	5.8	5.7	6.7	7.0
			4.8	5.2	5.0	4.9	5.0	5.1
		10	10.1	10.2	11.1	11.3	12.0	12.7
			10.0	9.8	9.9	10.2	10.0	9.9
10,10,15, 15,20,20	1,1,2, 2,3,3	1	1.1	1.0	1.2	1.3	1.4	1.5
			2.1	2.0	1.9	1.8	1.8	1.9
		5	4.8	5.1	5.4	5.3	6.9	6.9
			6.5	6.6	6.7	6.8	6.8	6.7
		10	10.3	10.1	10.8	11.3	12.3	12.6
			11.7	11.6	11.7	11.6	11.6	11.4
	3,3,2, 2,1,1	1	1.4	1.3	1.7	1.8	2.6	2.7
			2.0	1.9	1.9	1.7	1.7	1.7
		5	5.6	5.4	5.8	6.1	7.7	8.2
			6.1	6.1	5.8	6.2	6.2	6.3
		10	10.8	10.3	10.8	11.4	13.2	13.7
			11.4	11.1	11.0	11.3	11.0	11.5
20,20,20, 20,20,20,20, 20,20,20	1,1,1.5, 1.5,2,2,2.5, 2.5,3,3	1	1.2	1.3	1.4	1.8	1.8	2.6
			2.0	1.9	1.8	2.0	2.0	1.9
		5	5.1	5.2	5.9	6.1	7.4	8.7
			7.0	6.9	6.6	6.5	6.5	6.7
		10	10.6	10.7	11.1	12.2	13.2	14.5
			11.7	11.8	11.8	11.7	11.9	11.7

2.5 ASYMPTOTIC DISTRIBUTION OF THE TRIMMED F AND W

By using the result of Bickel [2] and the method of proof of Ponnapalli [21] to show that W is asymptotically distributed as chi-square, we shall derive the asymptotic distribution of the trimmed F and W.

2.5.1 Asymptotic Distribution of the Trimmed Mean

From theorem 3.1 of Bickel [2], we have

$$\sqrt{h}(\bar{X}_{tg} - \mu) \longrightarrow N(0, \sigma_g^2) \text{ in law as } n \longrightarrow \infty$$

where $\sigma_g^2 = [2gx_g^2 + \int_{x_g}^{x_{1-g}} t^2 dF(t)]/(1 - 2g)$, $F(x - \mu)$ is the underlying distribution function and x_g and x_{1-g} are 100g-th, 100(1 - g)-th percentiles of F(x), respectively. F(x) satisfies the following regularity conditions:

1. It is symmetric about 0.
2. It is absolutely continuous with respect to Lebesque measure.
3. It possesses a density which is continuous and strictly positive on $\{x: 0 < F(x) < 1\}$.

Using the above theorem we will derive the asymptotic distributions of $F_t(g)$ and $W_t(g)$ under those distributions which satisfy the regularity conditions.

2.5.2 Asymptotic Distribution of the Trimmed F

Here we assume that the distribution function $F_i(x)$ of i-th group is equal to $F(x - \mu_i)$, i = 1, 2, ... , c. Moreover, we assume that σ_g^2 is finite. Let $Y_i = \sqrt{h_i}\,\bar{X}_{itg}$ and $A = (a_{ij})$ be an orthonormal matrix with the last row $(\sqrt{h_1/H}, \sqrt{h_2/H}, \ldots , \sqrt{h_c/H})$. Consider the transformation Z = AY where $Z^t = (Z_1, Z_2, \ldots , Z_c)$, $Y^t = (Y_1, Y_2, \ldots , Y_c)$. Then,

$$\sum_{i=1}^{c} h_i(\bar{X}_{itg} - \bar{X}_{tg})^2 = \sum_{i=1}^{c} Y_i^2 - (\sum_{i=1}^{c}\sqrt{h_i/H}\, Y_i)^2 = \sum_{i=1}^{c} Z_i^2 - Z_c^2 = \sum_{i=1}^{c-1} Z_i^2.$$

Since Y_i's are independent and asymptotically normal, Z_i's are also independent and asymptotically normal with mean $E(Z_i) = \lambda_i = \sum_j a_{ij}\sqrt{h_j}\,\mu_j$ and variance σ_g^2 by the Mann-Wald theorem [17]. Again by the same theorem,

$$\sum_{i=1}^{c} h_i(\bar{X}_{itg} - \bar{X}_{tg})^2/\sigma_g^2 = \sum_{i=1}^{c-1} Z_i^2/\sigma_g^2$$

is asymptotically distributed as χ^2_{c-1} with noncentrality parameter $\phi = [(\lambda^t\lambda - \lambda_c^2)/\sigma_g^2]^{1/2}$ where $\lambda^t = (\lambda_1, \lambda_2, \ldots, \lambda_c)$. After some algebraic manipulations, we have

$$\phi = [\sum_{i=1}^{c} h_i(\mu_i - \bar{\mu})^2/\sigma_g^2]^{1/2}$$

where $\bar{\mu} = \sum_i h_i\mu_i/H$. Furthermore,

$$S_{iwg}^2 = SSD_{iwg}/(h_i - 1) \longrightarrow \sigma_g^2 \text{ in probability}$$

and thus, by Slutsky's theorm [5], as $n_1, n_2, \ldots, n_c \longrightarrow \infty$ with $\lim_{N\to\infty} n_i/N \neq 0$, $i = 1, 2, \ldots, c$, we have

$$\sum_{i=1}^{c} SSD_{iwg}/(H - c) \xrightarrow{p} \sigma_g^2.$$

Hence,

$(c-1)F_t(g) \longrightarrow \chi^2_{c-1}$ in law.

It can be shown that $\phi \longrightarrow \infty$ as $N \longrightarrow \infty$ under any alternative and hence that the test is consistent.

2.5.3 Asymptotic Distribution of the Trimmed W

Suppose that $F_i(x - \mu_i)$ is the distribution function of i-th group and $F_i(x)$ satisfies the regularity conditions given in Subsection 2.5.1, i = 1, 2, ... , c. Let $\sigma^2_{ig} = [2x^2_g + \int_{x_g}^{x_{1-g}} t^2 dF_i(t)]/(1-2g)$. Then,

$$\sqrt{h_i}(\bar{X}_{itg} - \mu_i) \longrightarrow N(0, \sigma^2_{ig}) \text{ in law,}$$

or,

$$\sqrt{h_i}(\bar{X}_{itg} - \mu_i)/\sigma_{ig} \longrightarrow N(0, 1) \text{ in law.}$$

If we put $\theta_i = h_i/\sigma^2_{ig}$, $\theta = \sum_i \theta_i$ and $Y_i = \sqrt{\theta_i}\,\bar{X}_{itg}$, then

$$\sum_{i=1}^{c} \theta_i(\bar{X}_{itg} - \check{X}_{tg})^2 = \sum_{i=1}^{c} \theta_i Y_i^2 - \Big(\sum_{i=1}^{c} \sqrt{\theta_i/\theta}\, Y_i\Big)^2$$

where $\check{X}_{tg} = \sum_i \theta_i \bar{X}_{itg}/\theta$. Transforming $Y^t = (Y_1, Y_2, \ldots , Y_c)$ to $Z^t = AY$ by an orthonormal matrix A such that the last row is $(\sqrt{\theta_1/\theta}, \sqrt{\theta_2/\theta}, \ldots , \sqrt{\theta_c/\theta})$, we have

$$V = \sum_{i=1}^{c} \theta_i(\bar{X}_{itg} - \check{X}_{tg})^2 = \sum_{i=1}^{c} Z_i^2 - Z_c^2 = \sum_{i=1}^{c-1} Z_i^2 .$$

Since Z_i's are independent and asymptotically normal, $V \longrightarrow \chi^2_{c-1}$ in law with noncentrality $\phi' = \sum_i \theta_i(\mu_i - \check{\mu})^2$ where $\check{\mu} = \sum_i (\theta_i/\theta)\mu_i$. By Slutsky's theorem $w_i \longrightarrow \theta_i$ and $w_i/w \longrightarrow \theta_i/\theta$ both in probability recalling that $w_i = h_i/S^2_{iwg}$ and $w = \sum_i w_i$. Clearly $U - V \longrightarrow 0$ in probability where $U = \sum_i w_i(\bar{X}_{itg} - \tilde{X}_{tg})^2$ and thus, $U \longrightarrow \chi^2_{c-1}$ in law with noncentrality ϕ'. Again by Slutsky's theorem, we see that the denominator of $W_t(g)$ converges to 1 in probability. Combining all the results obtained above, we get

$$(c - 1)W_t(g) \longrightarrow \chi^2_{c-1} \text{ in law}$$

as $n_i \longrightarrow \infty$ with $\lim_{N\to\infty} n_i/N \neq 0$, $i = 1, 2, \ldots, c$. The consistency of $W_t(g)$ follows from the fact that $\phi' \longrightarrow \infty$ as $N \longrightarrow \infty$ under any alternative.

Chapter III

SINE-WAVE STATISTICS FOR MULTI-SAMPLE LOCATION PROBLEMS

3.1 THE SINE-WAVE M-ESTIMATOR

Let $X_1, X_2, \ldots, X_n$ be a sample of size n. In Chapter 1, we gave the definition of the sine-wave M-estimator as the solution for T of the equation

$$\sum_{j=1}^{n} \psi[(X_j - T)/k] = 0 \qquad (1)$$

where $\psi(z) = \sin(z)$ for $|z| < \pi$ and 0 elsewhere. This M-estimator is not scale invariant. In order to make the estimator scale invariant, we solve the following equation instead of (1);

$$\sum_{j=1}^{n} \psi[(X_j - T)/(kA)] = 0 \qquad (2)$$

for some scale estimator A [12]. Equation (2) can be solved for T iteratively. Choosing for A the median of the absolute deviations of the sample from its median M, that is, $A = \text{median}\{|X_j - M|\}$, the one step of a Newton-Raphson iteration, starting at M, toward the implicit solution of (2) is given by

$$T = M + kA \tan^{-1}\left[\sum_{|Z_j|<\pi} \sin(Z_j) / \sum_{|Z_j|<\pi} \cos(Z_j)\right],$$

where $Z_j = (X_j - M)/(kA)$, $j = 1, 2, \ldots, n$, and Z_j's are regarded as angles in radian. The arctangent speeds convergence and essentially achieves it in one step [8].

Now define

$$S^2 = n(kA)^2 \sum_{|Z_j|<\pi} \sin^2(Z_j) / \left[\sum_{|Z_j|<\pi} \cos(Z_j)\right]^2.$$

This is a finite sample evaluation of the asymptotic variance formula for an M-estimator [8], which will serve as the matching variance estimator of T.

The parameter k, called tuning factor, determines T and S^2 and their behaviour. When k is large, T acts like the sample mean, but for small k, T tends to be robust against long-tailedness. In the Princeton study [1], they used k = 2.1 to define T which was denoted by AMT.

3.2 ONE-SAMPLE INTERVAL AND TEST PROCEDURE BASED ON T AND S^2

In 1973, Gross [8] proposed a confidence interval procedure for location based on T and S^2 with k = 2.1 which is robust for symmetric long-tailed distributions. He defined the 95% confidence interval (he called this the wave interval) as

$$(T - tS/\sqrt{n},\ T + tS/\sqrt{n})$$

where t is approximated by the 97.5% point of Student's t distribution with $(n - 1)/2$ degrees of freedom. He asserted that this interval covers the location parameter of the underlying distribution with at least 95% confidence when $n \geq 8$.

In 1976, Gross [10] evaluated 25 interval estimators including the wave interval estimator with $k = 1.8$, 2.4 under normal and long-tailed symmetric distributions. These wave interval procedures performed quite well in almost all the situations considered in that paper.

In 1980, Lim [16] used T and S^2 with $k = 2.1$ to modify the sequential test procedure. He first investigated the approximate behaviour of the t-like statistic $\sqrt{n}T/S$ (we shall call this sine-wave t) under normality and concluded that it can be well approximated by Student's t distribution with $(n - 1)/2$ degrees of freedom for not too small n. For small n, the distribution of the sine-wave t has slightly longer tail than that of the Student's t. Based on this, he presented the robust sequential t*.

3.3 SINE-WAVE F FOR EQUAL VARIANCES

3.3.1 Formulation of the Sine-Wave F

Our approach to derivation of a robust statistic for multi-sample ($c \geq 2$) location testing problems with equal variances using the sine-wave M-estimator is similar to the one-sample case. That is, we replaced the sample mean and variance in the usual F statistic with the sine-wave M-estimator and its matching variance estimator and then tried to approximate the distribution of the modified statistic by an F distribution by adjusting the degrees of freedom. The reasons for approximating by a traditional tabulated distribution rather than the exact one are twofold: first, the exact distribution is very difficult to obtain analytically, second, we do not need a special table of critical values in order to use this statistic.

However, the simple adjustment of degrees of freedom as in the one-sample case does not yield a good approximation when sample sizes are small or moderate. Consequently, we multiply the statistic by a function of N and then adjust the degrees of freedom accordingly. In the preliminary investigation, we found that the multiplier should be an increasing function of N with limit 1 and also a function of c. In this way we derived the sine-wave F statistic as follows:

$$F_S(k) = \frac{\sum_{i=1}^{c} n_i (T_{ik} - \bar{T}_k)^2 / (c - 1)}{\sum_{i=1}^{c} (n_i - 1) S_{ik}^2 / (N - c)} \, r_k(N)$$

where T_{ik} = the location M-estimator of i-th group,

$\bar{T}_k = \sum_i n_i T_{ik} / N$,

S_{ik}^2 = the matching variance estimator of T_{ik},

$r_k(N) = (1 - ca_k/N)^2$,

a_k = a constant depending on k.

We chose k = 1.8, 2.1 and 2.4, with the hope that $F_S(k)$ with k = 2.4 performs well for those distributions close to normal, $F_S(k)$ with k = 2.1 for moderately long-tailed distributions and $F_S(k)$ with k = 1.8 for very long-tailed ones.

The distribution of $F_S(k)$ can be well approximated by an F distribution with $(c - 1, f_k)$ degrees of freedom where $f_k = d_k(N - c)$ and d_k is a constant depending on k. By trial and error, we obtained values of a_k and d_k which give a good approximation as follows:

k	1.8	2.1	2.4
a_k	0.8	0.7	0.65
d_k	0.45	0.5	0.6

3.3.2 Approximate Behaviour of the Sine-Wave F under Normality

Monte Carlo technique was again employed to study the approximate behaviour of $F_S(k)$ under normality. The sampling experiment was performed in a similar way as the trimmed F.

Table 3 shows the results of the experiment. We see that the approximation of $F_S(k)$ by an F distribution with $(c - 1, f_k)$ degrees of freedom is adequate for all cases considered except when $c = 6$, $n_i = 7$, $i = 1, 2, \ldots, 6$. When c is large and sample sizes are small, $F_S(k)$ tends to have longer tails than F with degrees of freedom given before. However, we are satisfied with the results for $n_i \geq 10$ for all $c = 2, 3, 4, 6$.

TABLE 3

Empirical significance levels of the sine-wave F under normality

	α								
	1			5			10		
Sample sizes	k			k			k		
(n_i)	1.8	2.1	2.4	1.8	2.1	2.4	1.8	2.1	2.4
7,7	1.1	1.0	1.1	4.8	4.9	5.0	9.4	9.5	9.6
10,10	0.9	0.9	1.0	4.8	4.8	4.9	9.7	9.7	10.0
20,20	1.0	1.0	1.0	5.0	4.8	4.9	9.9	9.5	9.7
10,20	0.9	0.8	0.7	4.2	4.4	4.6	9.9	9.5	9.6
7,7,7	1.0	1.0	1.1	4.9	5.1	5.3	9.5	9.6	9.9
10,10,10	1.0	1.1	1.3	5.3	5.3	5.2	10.4	10.2	10.6
20,20,20	0.9	0.9	1.0	5.1	4.7	4.8	10.3	9.8	10.0
10,15,20	1.1	1.0	1.1	5.1	5.0	5.2	10.0	9.9	9.7
7,7,7,7	1.2	1.0	1.0	5.2	5.1	5.2	10.4	10.3	10.1
10,10,10,10	1.0	1.0	1.0	5.1	5.0	5.0	10.0	10.0	9.9
20,20,20,20	0.9	0.8	0.8	4.7	4.6	4.5	10.1	9.9	10.1
10,15,15,20	0.8	0.8	0.7	4.5	4.5	4.5	9.8	9.4	9.4
7,7,7,7,7,7	1.3	1.2	1.3	5.4	5.4	5.8	10.8	10.9	11.0
10,10,10,10,10,10	0.9	0.9	0.8	5.0	5.1	5.0	10.3	10.3	10.1
20,20,20,20,20,20	1.0	1.0	1.0	5.2	5.0	4.8	10.2	9.8	9.7
10,10,15,15,20,20	1.0	1.0	1.1	5.0	4.7	4.7	10.0	9.9	9.7

3.4 SINE-WAVE STATISTICS FOR UNEQUAL VARIANCES

Here again, we modify Welch's [29] and Brown and Forsythe's [4] statistics using T and S^2 to obtain statistics which will provide protection against heterogeneity of group variances and long-tailedness. Our course of modification is guided by the experience obtained in the previous section.

3.4.1 The Sine-Wave W

We define the sine-wave W by

$$W_S(k) = r_k(N) \frac{\sum_{i=1}^{c} w_i (T_{ik} - \tilde{T}_k)^2 / (c - 1)}{1 + [2(c - 2)/(c^2 - 1)] \sum_{i=1}^{c} (1 - w_i/w)^2 / (n_i - 1)}$$

where $w_i = n_i/S_{ik}^2$,

$w = \sum_i w_i$,

$\tilde{T}_k = \sum_i w_i T_{ik} / w$.

We conjectured that $W_S(k)$ is approximately F distributed with $(c - 1, f_{wk})$ degrees of freedom where

$$1/f_{wk} = [3/(c^2 - 1)] \sum_{i=1}^{c} (1 - w_i/w)^2 / [d_k(n_i - 1)].$$

The notations which appeared in the previous section have the same meanings as in that section.

3.4.2 The Sine-Wave F*

The modification of F* is

$$F_S^*(k) = r_k(N) \frac{\sum_{i=1}^{c} n_i (T_{ik} - \bar{T}_k)^2}{\sum_{i=1}^{c} (1 - n_i/N) S_{ik}^2}.$$

The notations used above is the same as in the previous two sections. As before, our conjecture is that $F_S^*(k)$ could be well approximated by the F distribution with $(c - 1, f_k^*)$ degrees of freedom where

$$1/f_k^* = \sum_{i=1}^{c} b_i^2/[d_k(n_i - 1)]$$

and

$$b_i = (1 - n_i/N)S_{ik}^2/[\sum_{j=1}^{c} (1 - n_j/N)S_{jk}^2].$$

3.4.3 Approximate Behaviour of $W_S(k)$ and $F_S^*(k)$ under Normality

The conjectured approximate behaviour of $W_S(k)$ and $F_S^*(k)$ was investigated by Monte Carlo simulations, which were performed in the same way as for $W_t(g)$, $F_t^*(g)$.

Table 4 shows the results. We see from this table that $W_S(k)$ is somewhat liberal for some cases with k = 1.8, for example, the case with (n_i) = (10, 10, 15, 15, 20, 20) and (σ_i) = (3, 3, 2, 2, 1, 1). For other cases with k = 2.1, 2.4, even though the fluctuation of empirical α's about the nominal values looks somewhat excessive, the approximation seems reasonable in the sense that the original statistics also have this much fluctuation. By the same token, we are content with the results of $F_S^*(k)$ for all three values of k.

We will not pursue the correction of the liberalism of $W_S(1.8)$. Instead we exclude this statistic in the evaluation.

TABLE 4

Empirical significance levels of $W_S(k)$ and $F_S^*(k)$ under normality

Sample sizes (n_i)	Standard deviations	Nominal α(%)	$W_S(k)$ 1.8	$W_S(k)$ 2.1	$W_S(k)$ 2.4	$F_S^*(k)$ 1.8	$F_S^*(k)$ 2.1	$F_S^*(k)$ 2.4
10,10,10,10	1,1,1,1	1	1.1	0.9	0.9	0.7	0.7	0.6
		5	5.5	5.2	5.3	3.9	4.1	4.4
		10	10.6	10.2	10.0	9.0	8.9	8.9
	1,2,2,3	1	1.6	1.5	1.4	1.4	1.3	1.4
		5	6.0	5.9	5.8	5.5	5.5	5.9
		10	11.9	11.1	11.3	10.0	9.9	10.1
15,15,15,15	1,1,1,1	1	1.3	1.1	1.0	0.9	0.9	0.8
		5	5.3	4.9	4.8	4.7	4.5	4.4
		10	10.1	9.5	9.6	9.6	9.4	9.4
	1,2,2,3	1	1.2	1.0	1.0	1.1	1.0	1.1
		5	5.1	4.9	4.7	5.0	5.1	5.1
		10	9.7	9.2	9.7	9.7	9.6	9.7
20,20,20,20	1,1,1,1	1	0.6	0.5	0.6	0.7	0.7	0.8
		5	4.5	4.2	4.5	4.5	4.5	4.5
		10	9.7	9.2	8.8	9.6	9.1	9.1
	1,2,2,3	1	1.2	1.1	1.0	1.6	1.6	1.6
		5	4.8	4.7	4.7	6.0	6.1	6.3
		10	9.7	9.4	9.4	10.6	10.5	10.7
10,15,15,20	1,1,1,1	1	1.3	1.2	1.1	0.9	0.8	0.9
		5	5.3	5.0	4.9	4.5	4.3	4.6
		10	10.4	10.0	9.9	9.6	9.5	9.4
	1,2,2,3	1	1.4	1.2	1.1	1.3	1.3	1.4
		5	5.1	5.0	4.9	5.2	5.1	5.4
		10	10.2	9.8	9.8	10.0	10.0	10.2
	3,2,2,1	1	1.7	1.6	1.4	1.4	1.4	1.4
		5	6.2	5.7	5.3	5.8	5.8	5.7
		10	10.9	10.7	10.4	10.1	10.4	10.4
10,10,10, 10,10,10	1,1,2, 2,3,3	1	2.3	2.1	1.9	1.3	1.3	1.4
		5	7.1	6.8	6.6	5.1	5.2	5.6
		10	12.7	11.8	11.8	9.4	9.7	10.2
20,20,20, 20,20,20	1,1,2, 2,3,3	1	1.4	1.1	0.9	1.6	1.5	1.5
		5	5.5	4.9	5.1	6.5	6.3	6.5
		10	10.7	10.2	10.3	11.3	11.4	11.5
10,10,15, 15,20,20	1,1,1, 1,1,1	1	2.1	1.8	1.7	1.2	1.0	1.0
		5	6.8	6.0	5.8	4.9	4.7	4.9
		10	12.9	11.6	11.4	10.4	10.0	10.1
	1,1,2, 2,3,3	1	1.4	1.1	1.1	1.5	1.6	1.6
		5	6.1	5.4	5.5	6.1	5.9	5.8
		10	11.8	10.7	10.5	10.9	11.0	11.2
	3,3,2, 2,1,1	1	2.8	2.3	2.1	1.6	1.6	1.8
		5	8.2	7.0	6.5	6.5	6.3	6.4
		10	13.7	12.5	12.1	11.9	11.7	11.7
20,20,20, 20,20,20,20, 20,20,20	1,1,1.5, 1.5,2,2,2.5, 2.5,3,3	1	1.9	1.5	1.3	1.5	1.4	1.5
		5	6.0	5.6	5.5	6.4	6.4	6.2
		10	11.5	10.5	10.0	11.3	11.0	11.0

Chapter IV

EVALUATION OF THE PROPOSED PROCEDURES

4.1 THE PERFORMANCE OF THE TRIMMED F AND THE SINE-WAVE F

The performance of $F_t(g)$ and $F_S(k)$ was evaluated under normal and seven other long-tailed symmetric distributions by Monte Carlo sampling experiments. The powers of these tests are compared among themselves and with the well known nonparametric Kruskal-Wallis test.

4.1.1 Distributions Used in the Experiment

In this experiment the distributions listed in Table 5 were used.

TABLE 5

Distributions used in the simulation

Tag	Distribution
NORMAL	Standard normal, N(0, 1)
10% 3N	Mixture of two normals, 0.9N(0, 1) + 0.1N(0, 9)
10% 10N	Mixture of two normals, 0.9N(0, 1) + 0.1N(0, 100)
D-EXP	Double exponential
25% 1/U	Mixture of normal and normal/uniform 0.75N(0, 1) + 0.25N(0, 1)/U[0, 1]
25% 3/U	Mixture of normal and normal/uniform 0.75N(0, 1) + 0.25N(0, 9)/U[0, 1]
ALL 1/U	Normal/uniform, N(0, 1)/U[0, 1]
CAUCHY	Cauchy, C(0, 1)

All these distributions were used previously in the Princeton study [1]. If we order them according to their tail-length, they can be arranged roughly as follows: NORMAL, 10% 3N, D-EXP, 10% 10N, 25% 1/U, 25% 3/U, ALL 1/U, CAUCHY.

4.1.2 Sampling Situations

A sampling situation is a combination of the underlying distribution, number of groups (c), sample sizes (n_i) and group means (μ_i).

Without loss of generality, we may assume for the null case that $\mu_i = 0$, $i = 1, 2, \ldots, c$. For an alternative hypothesis, we assume that μ_i's are equally spaced. We define

$$\phi = [\sum_{i=1}^{c} n_i(\mu_i - \bar{\mu})^2 / (c\sigma^2)]^{1/2}$$

where $\bar{\mu} = \sum_i n_i\mu_i/N$. ϕ is related to the noncentrality parameter δ by $\delta^2 = c\phi^2$. Further assuming that $0 = \mu_1 \leq \mu_2 \leq \ldots \leq \mu_c$, we have $\mu_i = (i - 1)d$ where

$$d = \sigma\phi\{cN/[N \sum_{i=1}^{c} i^2 n_i - (\sum_{i=1}^{c} in_i)^2]\}^{1/2}$$

is the distance between adjacent means. This does not mean that the alternative is necessarily an ordered one. Ordering of the μ_i's here is purely a matter of convenience because a different ordering with equally spaced μ_i's and with the same ϕ value does not change the power of the test statistics to be evaluated.

For those distributions whose second moments do not exist, we follow Randles and Hogg [22] and define σ as the solution of $F(\sigma) = 0.8413 = \Phi(1)$ where F is the distribution function of the underlying distribution and Φ is that of the standard normal distribution.

The values of ϕ used were $\phi = 0(0.5)2.0$ and the number of groups were $c = 4$ and 6. The sample sizes we considered were $(n_i) = (10, 10, 10, 10)$, (20, 20, 20, 20), (10, 15, 15, 20) for $c = 4$ and $(n_i) = (20, 20, 20, 20, 20, 20)$, (10, 10, 15, 15, 20, 20) for $c = 6$. Thus, all together 200 different sampling situations were considered.

4.1.3 Sampling Method

In order to generate a random sample $\{x_j\}$ of size n from those distributions given in Section 4.1.1, we used the IMSL random number generators. For the mixtures of two normals, the uniform random number generator GGUBS and the normal random number generator GGNPM were used. First, n uniform[0, 1] random numbers $\{u_j\}$ by GGUBS and n normal(0, 1) random numbers $\{y_j\}$ by GGNPM were generated. Set $x_j = y_j$ if $u_j < 1 - \gamma$ and $x_j = \sigma_m y_j$ otherwise, where γ is the mixing proportion and σ_m is the standard deviation of the other normal distribution to be mixed. For 10% 3N, $\gamma = 0.1$ and $\sigma_m = 3$; and for 10% 10N, $\gamma = 0.1$ and $\sigma_m = 10$. The normal distribution can be thought of as a special case obtained by setting $\gamma = 0$.

Generation of a random sample from the double exponential is done by using GGUBS and the exponential(1) random number generator GGEXN. As before, we generated n u's by GGUBS and n y's by GGEXN, and then set $x_j = y_j$ if $u_j < 0.5$ and $x_j = -y_j$ otherwise.

For 25% 1/U, 25% 3/U and ALL 1/U, we used again GGUBS and GGNPM in the following way: 2n random U[0, 1] numbers $\{u_{1j}\}$, $\{u_{2j}\}$ by GGUBS and n random N(0, 1) numbers $\{y_j\}$ by GGNPM were generated, and then set $x_j = y_j$ if $u_{1j} < 1 - \gamma$ and $x_j = \sigma_m y_j / u_{2j}$ otherwise, where γ is the mixing proportion and σ_m is the parameter in $N(0, \sigma_m^2)/U[0, 1]$. For 25% 1/U, $\gamma = 0.25$ and $\sigma_m = 1$; for 25% 3/U, $\gamma = 0.25$ and $\sigma_m = 3$; and $\gamma = 1$ and $\sigma_m = 1$ for All 1/U.

A random sample from the Cauchy distribution was obtained directly using the Cauchy(0, 1) random number generator GGCAY.

For each sampling situation described in Section 4.1.2, a set of n random numbers were generated in the way described above and assigned to i-th group, i = 1, 2, ... , c. Each number assigned to i-th group was increased by (i - 1)d in order that the i-th group has mean $\mu_i = (i - 1)d$. The $F_t(g)$, $F_s(k)$ and Kruskal-Wallis statistics were computed and checks were made to see if the null hypothesis was rejected. This procedure was repeated 5,000 times. The empirical power of each test is the proportion of times that test statistic falls into the rejection region with α = 1, 5, 10%.

The levels of trimming used were 100g = 0(5)25 for all situations. In the case of $F_s(k)$, k = 1.8(0.3)2.4 were used for (n_i) = (20, 20, 20, 20), (10, 10, 15, 15, 20, 20), and k = 2.1 for (n_i) = (10, 10, 10, 10), (10, 15, 15, 20), (20, 20, 20, 20, 20, 20).

Approximate critical values of $F_t(g)$ and $F_s(k)$ were obtained from the F distribution as described in Chapter 3 using the IMSL routine MDFI of the inverse F distribution function which gives percentage points even for nonintegral degrees of freedom.

Since the critical values of the Kruskal-Wallis test are not tabulated for $n_i \geq 10$ for any number of groups, we used the chi-square approximation to the statistic which is commonly used. The statistic is defined by

$$KW = 12/[N(N+1)] \sum_{i=1}^{c} [R_i - n_i(N+1)/2]^2/n_i$$

where R_i is the rank sum of the i-th group in the pooled sample. When n_i's are large, KW is distributed approximately as chi-square with c - 1 degrees of freedom [15]. Here again we used the IMSL routine MDCHI of the inverse chi-square distribution function to obtain critical points.

4.1.4 Accuracy of the Monte Carlo Simulation

Tiku [25] tabulated the power of the F test under normality. Using his table, we can get a rough idea about the accuracy of our Monte Carlo simulations. Table 6 shows the comparison of the empirical power of the F-test obtained from our Monte Carlo sampling experiment with the theoretical power obtained from Tiku's table using the method of interpolation suggested by him for a few cases, $(n_i) = (10, 10, 10, 10)$, $(20, 20, 20, 20)$ and $\alpha = 1, 5\%$.

For the Monte Carlo line in Table 6, the first number is the empirical power in percent and the second number is the standard error $100\sqrt{\hat{p}(1-\hat{p})/5000}\ \%$. This table shows that our Monte Carlo results are reasonably accurate and within two standard errors from the theoretical results for the cases given in the table.

TABLE 6

Comparison of the empirical power of F with the theoretical one

$(n_i) = (10, 10, 10, 10)$

Alpha	Method	ϕ = 0.0	0.5	1.0	2.0
1	Tiku	1.0	2.91	13.28	73.0
	Monte Carlo	1.1 ± 0.15	2.8 ± 0.23	12.6 ± 0.47	73.1 ± 0.63
5	Tiku	5.0	10.85	32.45	90.36
	Monte Carlo	4.9 ± 0.31	10.7 ± 0.44	31.5 ± 0.66	90.4 ± 0.42
		$(n_i) = (20,$	$20,$	$20,$	$20)$
1	Tiku	1.0	3.11	14.82	77.78
	Monte Carlo	0.9 ± 0.13	3.1 ± 0.25	15.8 ± 0.52	78.6 ± 0.58
5	Tiku	5.0	11.21	34.19	92.05
	Monte Carlo	5.0 ± 0.31	11.0 ± 0.44	34.5 ± 0.67	92.5 ± 0.37

4.1.5 Results

Tables 10 - 17 show detailed results of this study. Some highlights from these tables will be presented for each distribution.

All the points we are going to make later are based on the general performance of the statistics. The ranking (Table 7) and the graphs (Figure 2) given at the end of this subsection, however, are based on a specific case of $(n_i) = (20, 20, 20, 20)$, $\alpha = 5\%$. These will give us a rough idea on the behaviour of each statistic. For the other cases under the same distribution, we may have slightly different rankings and/or graphs but the discrepancy is only minor.

Note also that in the power comparisons, no adjustment was made in the empirical alphas to make them equal because the statistics will usually be used in practice without adjustment. All statistics ($F_t(g)$

with $100g = 0(5)25$, $F_S(k)$ with $k = 1.8(0.3)2.4$ and KW) were computed for $(n_i) = (20, 20, 20, 20)$ and $(10, 10, 15, 15, 20, 20)$. For other sample sizes, $F_S(1.8)$ and $F_S(2.4)$ were excluded because we expected a similar performance. In all cases we considered, the same set of samples was used in the calculation of each statistic.

In terms of the ranking, we use symbols which have the following meaning:

$A \geq B$ means that A is more powerful than B and the difference in power (%) is less than 2% for all ϕ.

$A > B$ means that A is more powerful than B and the greatest difference in power (%) is between 2 to 10% for some ϕ.

$A >> B$ means that A is more powerful than B and the greatest difference in power (%) is larger than 10% for some ϕ.

There are a few cases in which the power curves of two statistics cross each other, that is, one has lower power than the other for small values of ϕ and higher power for larger values of ϕ. In these cases, our judgement will be based on their performance for larger values of ϕ.

Under Normality (Table 10)

1. As expected, the F statistic is the best among all.
2. There is a slight loss of power in $F_t(g)$. The loss increases as g increases. (See Figure 2-(1).)

3. The chi-square approximation to KW is slightly conservative when $\alpha = 1$, 5%. This is true under other distributions also. (See Point 7 in Section 4.1.6.)

4. Among the $F_S(k)$'s, the order of performance is $F_S(2.4) > F_S(2.1) > F_S(1.8)$ but the differences are not great.

5. The trimmed F with a small value of g (≤ 0.1) is doing very well in this case and KW and $F_S(2.4)$ performed similarly.

Under 10% 3N (Table 11)

1. Among the trimmed F's, $F_t(0.1)$ is the best. The power of $F_t(g)$ increases considerably as g changes from 0 to 0.05 and increases slowly afterwards reaching the highest point at g = 0.1 and then decreases slowly. (See Figure 2-(2).)

2. Among the $F_S(k)$'s, the best is $F_S(2.4)$ but it is only slightly superior to $F_S(2.1)$ which is just a little better than $F_S(1.8)$. $F_S(k)$ seems to be a little conservative when sample sizes are small and equal.

3. The performance of $F_t(0.1)$ and KW are very close (see Figure 2-(2)) except when $\alpha = 1\%$. In this case, KW is slightly conservative and the power is slightly lower than that of $F_t(0.1)$. For all k, $F_S(k)$ is somewhat inferior to $F_t(0.1)$ and KW. But the performance of all statistics except F is not much different. The F statistic is inferior to all other statistics except in the case of $(n_i) = (10, 10, 10, 10)$ at lower values of ϕ where F is slightly better than the worst.

Under 10% 10N (Table 12)

1. Among the trimmed F's, $F_t(0.2)$ is the most powerful. $F_t(g)$'s with $0 \le g \le 0.1$ are quite conservative. It is worthwhile to mention the empirical alphas for this case in detail since it is not expected. When sample sizes are equal, the empirical alpha of $F_t(g)$ decreases in some cases for small values of g and then increases for large values of g. For unequal sample sizes, the empirical alpha oscillates while increasing as g increases. (See Point 2 in Section 4.1.6.) The power of $F_t(g)$ increases sharply as g changes from 0 to 0.15, but very slowly for $0.15 \le g \le 0.2$. The power reaches its highest point in the neighborhood of $g = 0.2$ and then decreases very slowly for $g > 0.2$. (See Figure 2-(3).)
2. $F_S(k)$'s are slightly conservative. Among them $F_S(2.1)$ seems to be the best performer even though $F_S(2.4)$ is a close competitor. The power of $F_S(1.8)$ is only slightly lower than that of $F_S(2.4)$.
3. All $F_S(k)$'s are quite superior to $F_t(0.2)$ which is slightly superior to KW. The F-test is far inferior to all the others.

Under D-EXP (Table 13)

1. Among the $F_t(g)$'s, $F_t(0.25)$ is the best for all cases except when $(n_i) = (10, 10, 10, 10)$ and $\alpha = 1\%$ in which case $F_t(0.15)$ is the best. The empirical α's are kept close to nominal values for all g. The power of $F_t(g)$ increases steadily as g increases. (See Figure 2-(4).)
2. For $F_S(2.1)$ and $F_S(2.4)$, it is hard to conclude which is more powerful. $F_S(1.8)$ is only slightly less powerful than $F_S(2.1)$ and $F_S(2.4)$. They are all somewhat conservative.

3. Contrary to the preceding distribution, the trimmed F and the Kruskal-Wallis test are more powerful than sine-wave F. In this case, $F_t(0.25)$ is the winner and KW is also doing well. The F-test is the worst except for a few cases in which it is slightly better than the worst.

Under 25% 1/U (Table 14)

1. The F statistic is very conservative and $F_t(0.05)$ is quite conservative in most situations. When $(n_i) = (10, 10, 10, 10)$, $F_t(0.05)$ is even more conservative than F although trimming corrects the conservatism of F substantially in most of the other cases. Among the trimmed F's, $F_t(0.2)$ seems to be the best even though $F_t(0.15)$ is a little more powerful than $F_t(0.2)$ in some cases. In any case, their performances are very close to each other. The power of $F_t(g)$ increases rapidly as g increases from 0 to 0.1. The power increase is then slower until it reaches its peak somewhere around $g = 0.15$ or 0.2. It then decreases slowly afterwards. (See Figure 2-(5).)
2. Among the sine-wave F's, $F_S(2.1)$ is the most powerful when $(n_i) = (20, 20, 20, 20)$, but $F_S(2.4)$ is the best for $(n_i) = (10, 10, 15, 15, 20, 20)$. For both cases, however, their performances are very close. $F_S(k)$'s are again slightly conservative.
3. The sine-wave F's dominate the other statistics. Some $F_t(g)$'s fall between $F_S(k)$'s and KW. The F-test is the worst among all. Note that under normality the power of F increases as the degrees of freedom increase with holding ϕ constant. But under this distribution we see the opposite phenomenon (compare the case of

$(n_i) = (10, 10, 10, 10)$ with that of $(n_i) = (20, 20, 20, 20)$). It is not so for the other statistics.

Under 25% 3/U (Table 15)

1. For $0 \leq g \leq 0.1$, $F_t(g)$'s are very conservative for equal sample sizes. For unequal sample sizes, they are a little less conservative. The power of F is extremely low and the power of $F_t(g)$ increases very rapidly as g increases from 0 to 0.2 and increases slowly afterwards. (See Figure 2-(6).) The graph shows that one can obtain even higher power if one chooses a g greater than 0.25.
2. Under this distribution, the sine-wave statistics show larger differences in power performance among themselves than before even though the differences are still not great. $F_S(1.8)$ is the most powerful one among the $F_S(k)$'s.
3. Again the sine-wave statistics do very well here. KW is considerably inferior to all $F_S(k)$'s and $F_t(g)$'s with high g. The F statistic is far inferior to all others. As in the case of 25% 1/U, the power of F decreases as n_i's increase with ϕ fixed. It is not so for the other statistics.

Under ALL 1/U (Table 16)

1. The trimmed F behaves in almost the same way as it does under 25% 3/U. Figure 2-(6) and (7) show the similar behaviour.
2. Among the sine-wave statistics, $F_S(1.8)$ is again the best. They are slightly conservative especially when sample sizes are equal.

3. In general, $F_t(0.25)$ is superior to the others. $F_S(1.8)$ which is the best among the $F_S(k)$'s is only slightly inferior to $F_t(0.25)$. KW performs poorly in this case. Again, the F-test is very much inferior to the other statistics.

Under CAUCHY (Table 17)

1. In this case also the trimmed F behaves similarly as in the two preceding distributions.
2. $F_S(1.8)$ stays on top among the sine-wave statistics. The slight conservatism of the $F_S(k)$'s persists.
3. The performance comparison is very similar to the preceding case.

TABLE 7

The ranking of some statistics for equal variances

$(n_i) = (20, 20, 20, 20)$ and $\alpha = 5\%$

NORMAL

$F > F_t(0.05) > F_t(0.1) \geq KW \geq F_S(2.4) > F_S(2.1) \geq$

$F_t(0.15) > F_S(1.8) \geq F_t(0.2) > F_t(0.25)$

10% 3N

$KW \geq F_t(0.1) \geq F_t(0.15) \geq F_S(2.4) \geq F_S(2.1) \geq F_t(0.2) \geq$

$F_t(0.05) \geq F_S(1.8) \geq F_t(0.25) > F$

10% 10N

$F_S(2.4) \geq F_S(2.1) \geq F_S(1.8) > F_t(0.2) \geq KW \geq F_t(0.25) \geq$

$F_t(0.15) > F_t(0.1) >> F_t(0.05) >> F$

D-EXP

$F_t(0.25) > F_t(0.2) \geq KW \geq F_t(0.15) > F_t(0.1) \geq F_S(2.4) \geq$

$F_S(2.1) \geq F_S(1.8) > F_t(0.05) > F$

25% 1/U

$F_S(2.1) \geq F_S(2.4) \geq F_S(1.8) \geq F_t(0.15) \geq F_t(0.2) \geq KW \geq$

$F_t(0.25) \geq F_t(0.1) >> F_t(0.05) >> F$

25% 3/U

$F_S(1.8) > F_S(2.1) \geq F_t(0.25) \geq F_S(2.4) > F_t(0.2) > KW >$

$F_t(0.15) >> F_t(0.1) >> F_t(0.05) >> F$

ALL 1/U

$F_t(0.25) \geq F_S(1.8) > F_S(2.1) \geq F_t(0.2) > F_S(2.4) > KW >$

$F_t(0.15) >> F_t(0.1) >> F_t(0.05) >> F$

CAUCHY

$F_t(0.25) > F_S(1.8) > F_S(2.1) \geq F_t(0.2) \geq F_S(2.4) > KW >$

$F_t(0.15) >> F_t(0.1) >> F_t(0.05) >> F$

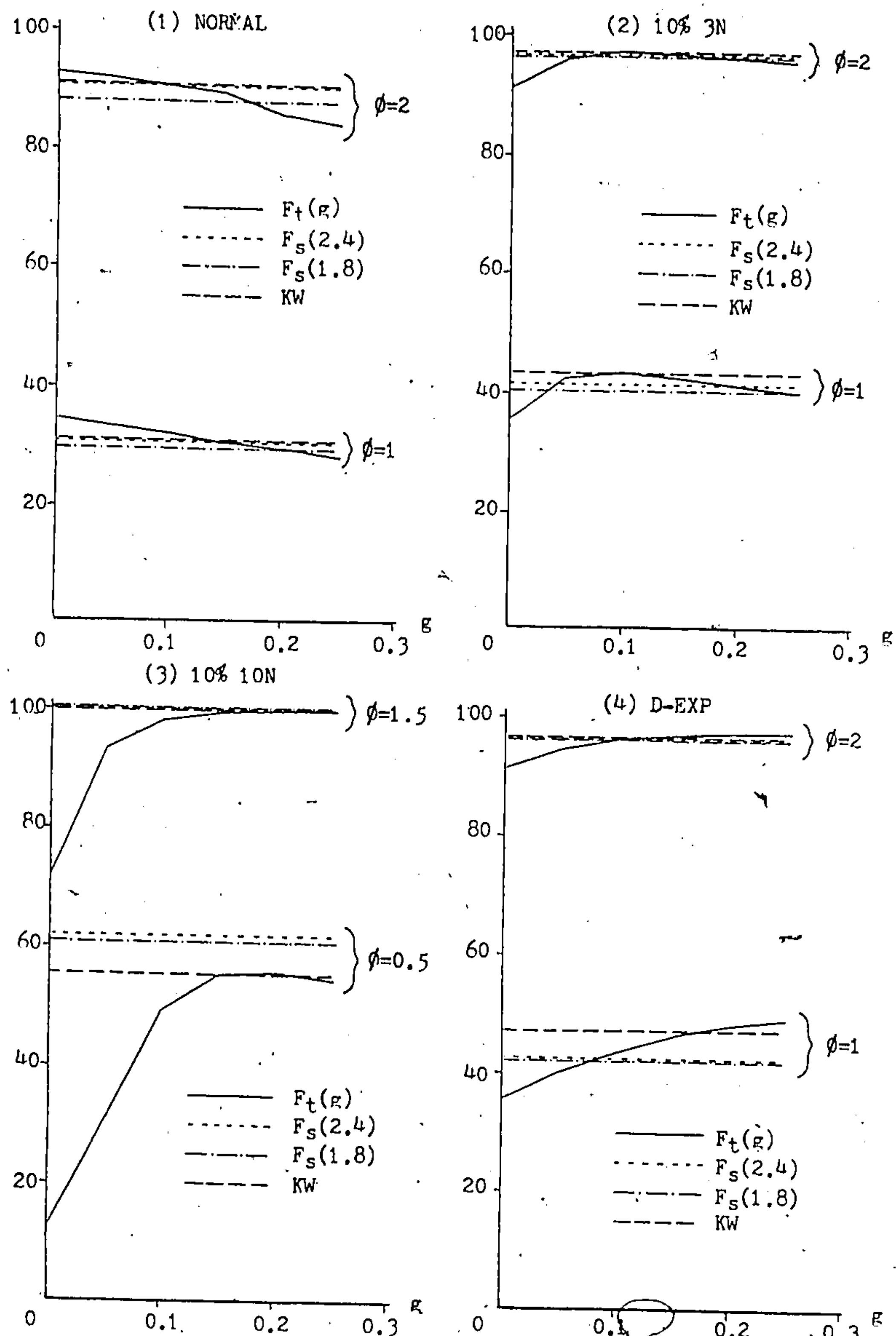
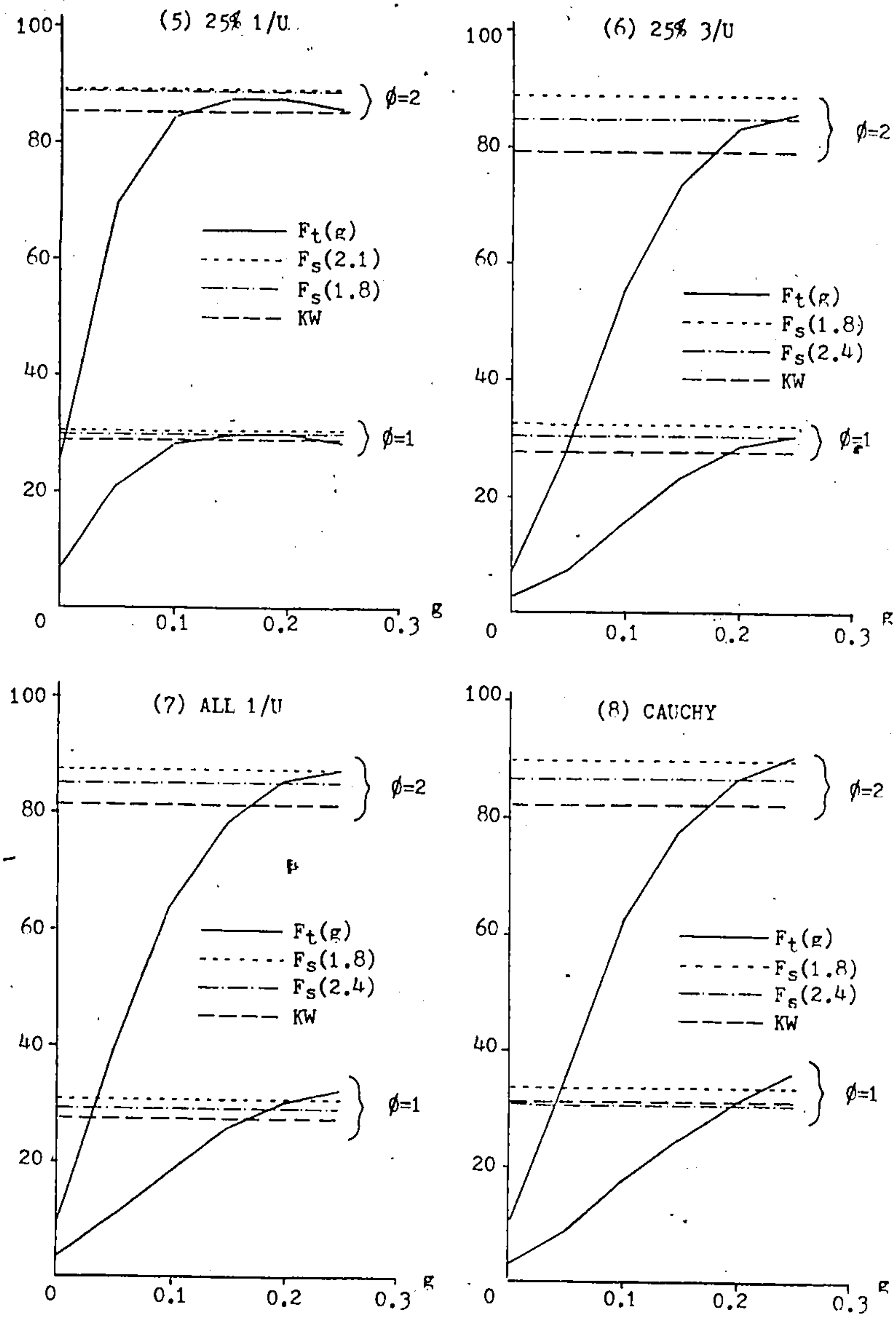


Figure 2: The power of $F_t(g)$ as a function of g, powers of the best and the worst $F_s(k)$ and KW at two values of ϕ based on (n_i) = (20, 20, 20, 20), α = 5%

Figure 2 cont'd



4.1.6 General Conclusions

Summarizing the results above, we have the following general conclusions. Recall that the order of distributions according to the tail-length is roughly

NORMAL < 10% 3N < D-EXP < 10% 10N < 25% 1/U < 25% 3/U < ALL 1/U < CAHCHY.

1. The F statistic is the worst among all statistics we considered under all situations in this study except in case of normality. Even under a normal distribution, other statistics are only slightly inferior to F. For long-tailed distributions such as 25% 1/U, 25% 3/U, ALL 1/U and CAUCHY, F is very conservative and has extremely low power. Also under these distributions, the power of F decreases as sample sizes increase with ϕ fixed. This is the opposite behaviour to that of F under normality. This may be one of the ill effects of heavy-tailedness on the F statistic. This phenomenon does not occur for the other statistics.
2. Under long-tailed distributions, $F_t(g)$ with a small trimming proportion is also conservative. As trimming proportion increases, we expect the conservatism to be corrected. This is shown to be true in general, but it does not occur in every single case and at every level of trimming. For example, under 10% 10N, $F_t(0.05)$ is even more conservative than F. In the neighborhood of the best g, however, the empirical alpha of $F_t(g)$ was kept close to the nominal value unless the underlying distribution is extremely long-tailed. The power of $F_t(g)$ for all levels of trimming is much higher than F under long-tailed distributions.

3. The best trimming proportion depends not only on the underlying distribution but also on alpha and sample sizes. Under the same parent distribution, the best $F_t(g)$ for one situation may be different from the best $F_t(g)$ for another situation. For example, under 10% 3N, $F_t(0.1)$ is the best when $\alpha = 1\%$ but $F_t(0.15)$ is the best when $\alpha = 5, 10\%$. However, their performances are very close.
4. Under NORMAL, 10% 3N, 10% 10N, D-EXP and 25% 1/U, the best $F_t(g)$ differs only slightly in power with the adjacent $F_t(g)$, which means that selecting a wrong value of g for these distributions cause only a little loss in power as long as it is close to the best g. Under extremely long-tailed situations such as 25% 3/U, ALL 1/U and CAUCHY, graphs (6) - (8) of Figure 2 indicate that $F_t(g)$ could achieve even higher power for some $g > 0.25$.
5. The sine-wave F has a tendency of being slightly conservative under long-tailed distributions. The performance of the three sine-wave statistic do not differ very much in any situations. As expected, $F_S(2.4)$ performed better than $F_S(1.8)$ and $F_S(2.1)$ under NORMAL, 10% 3N and D-EXP but $F_S(1.8)$ did best under the very long-tailed distributions like 25% 3/U, ALL 1/U and CAUCHY. Under 10% 10N and 25% 1/U it is hard to decide which is better between $F_S(2.4)$ and $F_S(2.1)$. Based on $(n_i) = (20, 20, 20, 20)$, and (10, 10, 15, 15, 20, 20) in which we have studied all three $F_S(k)$'s, we obtained the following table:

TABLE 8

The best and the worst $F_S(k)$

Distribution	Best k	Worst k	Max. difference*
NORMAL	2.4	1.8	6.7
10% 3N	2.4	1.8	3.8
10% 10N	2.1 or 2.4	1.8	1.5
D-EXP	2.4	1.8	2.3
25% 1/U	2.1 or 2.4	1.8	2.3
25% 3/U	1.8	2.4	5.3
ALL 1/U	1.8	2.4	4.4
CAUCHY	1.8	2.4	5.7

*The maximum absolute difference in power (%) between the best and the worst $F_S(k)$ for $\phi = 0(0.5)2.0$.

In all distributions, the performance of $F_S(2.1)$ is either the best or between the best and the worst. The difference in power between $F_S(2.1)$ and the best is very small. Therefore, if we have to choose only one value of k, we should choose $k = 2.1$.

6. Under long-tailed distributions, F, $F_t(g)$ and $F_S(k)$ are less conservative when sample sizes are unequal than when they are equal. This can be explained by an indicator which measures the approximate effect of nonnormality on F [3]. It is defined by $B = \beta_2 b/2N$, where β_2 is the kurtosis of the underlying distribution and

$$b = 2\{N(N+1)(\sum_{i=1}^{c} 1/n_i - c^2/N)/[2(c-1)(N-c)] - 1\}$$

which is bounded by -2 and N - 1. When B is close to zero, the performance of F is close to normal theory. If B is negative, F will be conservative; if B is positive, F will be liberal. When sample sizes are equal, b attains the value of its lower bound -2. When sample sizes are unequal, b increases from -2 as the degree of imbalance in sample sizes increases. The following table shows that the values of β_2, b and B with (n_i) = (10, 10, 15, 15, 20, 20) for NORMAL, 10% 3N, 10% 10N and D-EXP which have finite kurtosis.

	Distribution			
	NORMAL	10% 3N	10% 10N	D-EXP
β_2	0	5.33	22.27	3
b	-1.35	-1.35	-1.35	-1.35
B	0	-0.040	-0.167	-0.023

When sample sizes are mildly imbalanced, b is closer to zero than when they are equal, and this makes B closer to zero. Therefore, the effect of nonnormality on F is reduced and thus F is less conservative. We expect a similar effect on $F_t(g)$. It is not recommended, however, to use trimmed F when sample sizes are extremely unbalanced because in this case B will be positive and F will be liberal and $F_t(g)$ with $g > 0$ will be even more liberal.

7. As a distribution-free statistic, KW keeps the empirical alpha reasonably close to the nominal value under all situations even though KW is slightly conservative at α = 1, 5% because we used the chi-square approximation to KW which is conservative at low levels of alpha [15].

Figure 2 illustrates Points (8) - (11).

8. Comparing the best performer among $F_t(g)$'s with the best among $F_S(k)$'s, the trimmed F is superior under NORMAL, 10% 3N, D-EXP, ALL 1/U and CAUCHY. On the other hand, the sine-wave statistic is more powerful to the trimmed F under 10% 10N, 25% 1/U and 25% 3/U.

9. The comparison of $F_S(k)$ with KW is as follows: under NORMAL, KW and $F_S(2.4)$, which is the best among $F_S(k)$'s, performed very closely. KW showed its strength over $F_S(k)$ under 10% 3N and D-EXP. For other distributions, the sine-wave statistics are superior to KW for all k = 1.8, 2.1, 2.4. Under very long-tailed distributions such as 10% 10N, 25% 3/U, ALL 1/U and CAUCHY, the best sine-wave statistic $F_S(1.8)$ is considerably more powerful than KW.

10. There is always at least one g such that $F_t(g)$ performs just as well as KW under 10% 3N and 10% 10N, and better than KW under the rest of the distributions.

11. To summarize points 8 - 10, we present the best 4 statistics for each distribution in Table 9 based on the results of (n_i) = (20, 20, 20, 20) and (10, 10, 15, 15, 20, 20) (recall that all the statistics were computed for these cases). In this table, KW appears only three times, under normality and moderately long-tailed distributions such as 10% 3N, D-EXP. $F_S(k)$ performed fairly well under long-tailed distributions whose tails are longer than that of 10% 3N or D-EXP. The trimmed F appears everywhere and they did well especially under normality and moderately long-tailed distributions such as D-EXP, 10% 3N (under

10% 3N, $F_t(0.1)$ and KW are very close), and under extremely long-tailed situations such as ALL 1/U and CAUCHY.

TABLE 9

The best 4 statistics

Distribution	1st	2nd	3rd	4th
NORMAL	F	$F_t(0.05)$	$F_t(0.1)$	KW
10% 3N	KW	$F_t(0.1)$	$F_t(0.15)$	$F_s(2.4)$
10% 10N	$F_s(2.4)$	$F_s(2.1)$	$F_s(1.8)$	$F_t(0.2)$
D-EXP	$F_t(0.25)$	$F_t(0.2)$	KW	$F_t(0.15)$
25% 1/U	$F_s(2.1)$	$F_s(2.4)$	$F_s(1.8)$	$F_t(0.15)$
25% 3/U	$F_s(1.8)$	$F_s(2.1)$	$F_t(0.25)$	$F_s(2.4)$
ALL 1/U	$F_t(0.25)$	$F_s(1.8)$	$F_s(2.1)$	$F_t(0.2)$
CAUCHY	$F_t(0.25)$	$F_s(1.8)$	$F_s(2.1)$	$F_t(0.2)$

4.1.7 Recommendations

It is difficult to give clear-cut recommendations partially because our investigation does not cover all cases and partially because a rigorous comparison is difficult to be made from empirical results. Also the preference of one statistic to another depends on robustness in validity and power performance as well as other criteria such as simplicity in computation and usage. Nonetheless, we give the following recommendations mainly based on robustness in validity and power performance.

1. Assuming we have a good knowledge on the tail-length of the underlying distribution, the trimmed F with a proper trimming proportion (including 0) is recommended under normal or close to normal and extremely long-tailed situations. A general rule on trimming proportion is that the longer the tail, the larger the

trimming proportion should be. The Kruskal-Wallis is recommended only when the parent distribution has tail length close to 10% 3N. For those distributions whose tails are close to that of 10% 10N or 25% 1/U, the sine-wave statistic with $k = 2.1$ or 2.4 should be used.

2. If we have only a rough idea about the tail-length, $F_t(0.1)$ or $F_s(2.4)$ or KW are recommended for moderately long-tailed situations and $F_t(0.25)$ or $F_s(1.8)$ are suggested for severely long-tailed situations.

3. In the case that we have no prior information at all about the tail-length of the parent distribution, $F_s(2.1)$ is recommended because of its remarkable overall performance.

TABLE 10

Powers of some statistics for equal variances under normality

Legend

TRF(100g): Trimmed F, $F_t(g)$
SWF(k) : Sine-wave F, $F_s(k)$
KW : Kruskal-Wallis

TABLE 10-(1)

SAMPLE SIZES= 10 10 10 10
DISTRIBUTION:NORMAL

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	1.1	2.8	12.6	38.6	73.1
	TRF(5)	1.1	2.7	12.1	37.6	71.9
	TRF(10)	1.1	2.7	11.0	33.9	66.8
	TRF(15)	1.0	2.4	10.7	32.2	64.6
	TRF(20)	1.1	2.5	9.8	28.1	57.8
	TRF(25)	1.0	2.6	9.4	26.4	54.4
	SWF(2.1)	1.1	2.3	8.4	27.0	59.6
	KW	0.7	1.9	9.3	30.8	65.5
5						
	TRF(0)	4.9	10.7	31.5	64.4	90.4
	TRF(5)	5.1	10.8	30.8	63.7	89.8
	TRF(10)	5.2	10.4	29.0	60.3	87.6
	TRF(15)	5.2	10.4	28.4	59.1	86.8
	TRF(20)	5.6	10.0	26.1	55.2	82.2
	TRF(25)	5.7	10.2	25.5	53.6	80.3
	SWF(2.1)	5.0	9.3	26.0	55.9	84.7
	KW	4.5	9.6	28.4	60.2	87.8
10						
	TRF(0)	10.3	18.7	45.2	77.1	94.8
	TRF(5)	10.2	18.7	44.3	76.4	94.4
	TRF(10)	9.9	17.7	41.5	73.8	93.3
	TRF(15)	10.3	17.7	40.7	72.5	92.8
	TRF(20)	10.5	17.4	38.3	68.6	90.5
	TRF(25)	11.1	18.2	37.8	67.7	89.3
	SWF(2.1)	10.1	16.6	38.1	70.6	91.6
	KW	10.0	17.5	42.2	74.1	93.6

TABLE 10-(2)

SAMPLE SIZES= 20 20 20 20
DISTRIBUTION: NORMAL

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.9	3.1	15.8	44.4	78.6
	TRF(5)	0.8	3.0	14.7	43.3	76.4
	TRF(10)	1.0	3.2	14.1	41.1	73.7
	TRF(15)	1.1	3.1	12.7	38.6	70.4
	TRF(20)	1.2	3.1	12.2	35.6	66.8
	TRF(25)	1.1	3.3	11.5	32.8	63.4
	SWF(1.8)	0.7	2.6	11.1	34.5	66.3
	SWF(2.1)	0.7	2.7	12.1	37.0	70.2
	SWF(2.4)	0.7	2.7	13.0	39.6	72.9
	KW	0.7	2.6	13.3	40.5	74.4
5						
	TRF(0)	5.0	11.0	34.5	68.8	92.5
	TRF(5)	4.9	10.6	33.3	67.5	91.8
	TRF(10)	4.7	11.0	32.1	64.7	90.5
	TRF(15)	4.8	10.7	30.4	62.7	89.1
	TRF(20)	5.4	10.7	29.7	60.2	86.9
	TRF(25)	5.6	10.5	28.1	57.4	84.2
	SWF(1.8)	4.6	9.7	29.2	60.5	87.7
	SWF(2.1)	4.4	9.9	30.4	62.8	89.4
	SWF(2.4)	4.6	10.0	31.6	64.4	90.5
	KW	4.4	10.4	31.9	65.6	90.5
10						
	TRF(0)	10.1	18.9	47.9	78.5	95.9
	TRF(5)	10.0	18.5	46.7	77.4	95.5
	TRF(10)	10.1	18.5	45.5	76.0	95.0
	TRF(15)	10.1	18.2	43.8	74.2	93.9
	TRF(20)	10.3	17.7	42.3	71.9	92.7
	TRF(25)	10.6	17.9	41.0	70.2	91.2
	SWF(1.8)	9.8	16.9	42.1	73.4	93.5
	SWF(2.1)	9.8	17.0	43.8	75.1	94.4
	SWF(2.4)	9.9	17.4	44.7	76.1	95.0
	KW	9.9	18.1	45.7	76.5	95.0

TABLE 10-(3)

SAMPLE SIZES= 10 15 15 20
DISTRIBUTION:NORMAL

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.8	2.9	15.0	42.0	76.1
	TRF(5)	0.9	2.8	14.7	40.3	74.5
	TRF(10)	1.0	3.0	13.8	37.5	71.1
	TRF(15)	1.0	2.8	12.9	35.2	67.2
	TRF(20)	1.2	2.9	12.2	31.6	62.9
	TRF(25)	1.1	2.9	11.7	29.3	59.1
	SWF(2.1)	1.0	2.5	11.3	32.0	65.4
	KW	0.7	2.6	12.2	35.5	69.9
5						
	TRF(0)	4.7	10.8	34.4	67.2	91.1
	TRF(5)	4.5	10.5	33.9	65.1	90.4
	TRF(10)	4.7	10.2	32.8	62.8	89.0
	TRF(15)	4.9	9.8	31.4	60.6	87.2
	TRF(20)	5.3	9.8	29.6	57.8	84.7
	TRF(25)	5.6	10.1	28.1	54.7	82.1
	SWF(2.1)	4.8	9.5	29.3	59.8	86.9
	KW	4.4	9.4	31.9	62.7	89.4
10						
	TRF(0)	9.7	18.9	47.1	78.3	95.4
	TRF(5)	9.4	18.4	46.1	77.5	94.8
	TRF(10)	9.6	17.9	45.1	75.7	93.9
	TRF(15)	9.9	17.6	43.6	73.5	93.2
	TRF(20)	9.8	17.6	41.9	70.6	91.7
	TRF(25)	10.7	18.0	39.9	67.8	90.2
	SWF(2.1)	9.4	17.2	43.2	73.1	93.0
	KW	9.4	18.0	44.8	75.8	94.1

TABLE 10-(4)

SAMPLE SIZES= 20 20 20 20 20 20
DISTRIBUTION: NORMAL

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.8	3.4	19.6	58.2	90.7
	TRF(5)	0.9	3.4	17.9	56.1	89.5
	TRF(10)	0.9	3.2	16.9	53.0	87.6
	TRF(15)	0.9	3.0	15.9	49.9	84.8
	TRF(20)	0.8	2.8	15.2	47.3	82.0
	TRF(25)	0.8	2.8	14.2	43.5	77.6
	SWF(2.1)	0.8	2.5	14.5	49.6	85.5
	KW	0.7	2.9	16.4	53.0	87.9
5						
	TRF(0)	4.9	12.8	40.8	80.2	97.3
	TRF(5)	5.0	12.4	39.4	78.2	97.0
	TRF(10)	4.4	12.0	37.8	76.5	96.3
	TRF(15)	4.8	11.7	36.3	74.5	95.4
	TRF(20)	5.0	11.3	34.9	71.7	94.5
	TRF(25)	4.9	11.2	32.5	68.7	92.7
	SWF(2.1)	4.5	10.9	36.5	74.6	95.7
	KW	4.6	11.7	38.6	77.3	96.5
10						
	TRF(0)	9.7	20.5	53.2	88.2	98.8
	TRF(5)	9.7	20.0	52.1	87.0	98.5
	TRF(10)	9.9	19.5	50.9	85.8	98.4
	TRF(15)	10.2	19.5	49.3	84.3	98.1
	TRF(20)	10.1	19.6	47.8	81.8	97.2
	TRF(25)	10.2	19.2	45.8	79.5	96.4
	SWF(2.1)	9.4	18.9	49.4	85.0	98.2
	KW	9.9	19.5	51.5	85.[illegible]	98.4

TABLE 10-(5)

SAMPLE SIZES= 10 10 15 15 20 20
DISTRIBUTION: NORMAL

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.7	3.7	19.1	58.1	89.4
	TRF(5)	0.6	3.6	18.6	55.9	87.7
	TRF(10)	0.6	3.8	16.5	52.3	85.0
	TRF(15)	0.8	3.6	15.7	49.2	82.4
	TRF(20)	0.7	3.7	14.6	44.6	78.2
	TRF(25)	1.0	3.3	13.3	41.9	74.6
	SWF(1.8)	0.7	3.2	13.0	43.4	78.4
	SWF(2.1)	0.6	3.4	14.4	47.4	81.4
	SWF(2.4)	0.6	3.5	15.5	50.1	84.2
	KW	0.6	3.2	15.1	50.2	85.0
5						
	TRF(0)	4.4	13.0	40.1	79.7	97.1
	TRF(5)	4.3	12.5	39.1	78.5	96.8
	TRF(10)	4.7	12.2	37.3	75.8	95.7
	TRF(15)	4.7	12.0	36.0	74.4	94.7
	TRF(20)	4.6	11.2	33.8	70.8	92.9
	TRF(25)	4.9	11.7	32.5	67.9	91.0
	SWF(1.8)	5.1	11.1	32.8	70.9	93.7
	SWF(2.1)	4.8	11.1	34.8	73.7	94.8
	SWF(2.4)	4.6	11.5	36.3	75.6	95.5
	KW	3.8	11.4	36.9	76.7	96.0
10						
	TRF(0)	9.2	21.3	54.3	87.9	98.6
	TRF(5)	9.1	20.8	52.7	87.0	98.5
	TRF(10)	9.2	19.7	50.6	85.1	98.2
	TRF(15)	9.6	19.4	49.1	83.9	97.7
	TRF(20)	9.6	19.0	47.2	81.5	96.6
	TRF(25)	10.2	19.5	46.2	79.2	95.7
	SWF(1.8)	9.5	19.3	46.5	81.6	97.2
	SWF(2.1)	9.4	19.8	48.6	83.7	97.9
	SWF(2.4)	9.4	20.0	49.8	85.0	98.2
	KW	8.7	19.8	50.9	86.0	98.3

TABLE 11

Powers of some statistics for equal variances under 10% 3N

Legend
TRF(100g): Trimmed F, $F_t(g)$
SWF(k) : Sine-wave F, $F_S(k)$
KW : Kruskal-Wallis

TABLE 11-(1)

SAMPLE SIZES= 10 10 10 10
DISTRIBUTION: 10% 3N

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.6	2.8	15.4	44.6	75.5
	TRF(5)	0.6	2.9	16.8	49.7	81.8
	TRF(10)	0.7	3.2	18.1	52.4	84.3
	TRF(15)	0.7	3.4	17.4	51.7	83.9
	TRF(20)	0.8	3.6	15.5	47.1	79.9
	TRF(25)	0.8	3.6	14.5	44.2	77.0
	SWF(2.1)	0.5	2.7	13.8	44.4	78.7
	KW	0.5	2.7	16.0	48.9	82.5
5						
	TRF(0)	4.3	11.7	35.3	68.7	89.8
	TRF(5)	4.5	12.1	38.4	74.0	94.0
	TRF(10)	4.5	12.6	39.9	76.0	95.3
	TRF(15)	4.4	13.1	39.8	76.5	95.3
	TRF(20)	4.5	13.3	37.7	73.6	94.2
	TRF(25)	4.8	13.7	37.2	72.0	93.2
	SWF(2.1)	3.7	11.4	36.0	72.3	94.2
	KW	3.8	13.0	40.0	76.1	95.2
10						
	TRF(0)	9.3	20.9	47.3	78.9	94.2
	TRF(5)	8.9	21.6	51.1	83.9	96.5
	TRF(10)	9.2	21.9	53.3	86.0	97.6
	TRF(15)	9.2	22.5	53.6	86.4	97.7
	TRF(20)	9.4	21.6	51.4	84.4	97.3
	TRF(25)	10.0	21.4	51.1	83.4	97.0
	SWF(2.1)	7.9	19.8	50.1	83.7	97.1
	KW	9.2	22.4	54.5	86.5	97.5

TABLE 11-(2)

SAMPLE SIZES= 20 20 20 20
DISTRIBUTION: 10% 3N

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.7	3.0	15.7	48.5	77.9
	TRF(5)	0.8	3.3	20.0	59.4	87.8
	TRF(10)	0.9	3.6	21.0	60.8	89.7
	TRF(15)	0.9	3.7	20.6	60.3	89.1
	TRF(20)	0.9	3.8	20.0	57.1	87.9
	TRF(25)	1.1	3.9	18.9	53.7	85.3
	SWF(1.8)	0.7	3.1	17.2	55.1	87.1
	SWF(2.1)	0.7	3.0	17.8	57.5	88.1
	SWF(2.4)	0.7	3.1	18.9	58.9	88.8
	KW	0.8	3.5	20.9	60.4	89.4
5						
	TRF(0)	4.4	11.4	35.6	70.4	91.0
	TRF(5)	4.6	12.2	42.2	79.2	95.9
	TRF(10)	4.9	13.3	43.3	80.8	97.1
	TRF(15)	5.0	13.4	42.9	80.5	96.9
	TRF(20)	5.4	13.6	41.8	79.8	96.3
	TRF(25)	5.4	13.3	40.1	78.0	95.6
	SWF(1.8)	4.0	12.2	40.2	79.4	96.4
	SWF(2.1)	4.1	12.3	41.2	80.2	96.8
	SWF(2.4)	4.0	12.3	41.5	80.2	96.9
	KW	4.6	13.3	43.5	80.9	97.1
10						
	TRF(0)	9.3	19.5	48.4	79.1	95.2
	TRF(5)	9.1	21.7	55.4	86.7	98.1
	TRF(10)	9.5	22.2	57.0	88.3	98.8
	TRF(15)	9.7	22.1	56.2	88.6	98.8
	TRF(20)	9.6	22.4	55.2	87.5	98.4
	TRF(25)	10.1	21.8	53.3	86.4	98.0
	SWF(1.8)	9.4	20.9	53.9	87.6	98.4
	SWF(2.1)	9.1	21.3	54.7	88.5	98.6
	SWF(2.4)	9.1	21.4	54.9	88.2	98.7
	KW	9.5	23.2	57.7	88.3	98.8

TABLE 11-(3)

SAMPLE SIZES= 10 15 15 20
DISTRIBUTION: 10% 3N

ALPHA	PHI 0.0	0.5	1.0	1.5	2.0
1					
TRF(0)	0.6	3.1	16.0	46.3	77.2
TRF(5)	0.6	3.7	19.5	54.9	86.0
TRF(10)	0.7	4.0	20.3	57.7	88.6
TRF(15)	1.0	3.9	20.2	56.9	88.1
TRF(20)	1.3	3.9	19.3	53.9	85.5
TRF(25)	1.3	3.6	18.7	50.9	82.8
SWF(2.1)	0.9	3.0	17.4	52.5	85.6
KW	0.6	3.4	19.1	55.7	87.5
5					
TRF(0)	4.2	11.7	36.3	69.1	90.5
TRF(5)	5.0	12.7	42.2	77.8	95.7
TRF(10)	4.7	12.8	43.2	80.3	96.7
TRF(15)	5.1	12.7	43.8	80.2	96.8
TRF(20)	5.1	12.6	41.5	78.5	95.8
TRF(25)	5.3	12.9	40.7	76.1	95.2
SWF(2.1)	4.3	11.5	40.3	77.8	96.3
KW	4.5	12.8	43.2	80.0	96.6
10					
TRF(0)	9.9	20.4	50.1	79.2	94.7
TRF(5)	10.3	21.7	55.9	85.6	98.0
TRF(10)	10.1	21.9	57.2	88.0	98.5
TRF(15)	10.6	22.3	57.2	87.9	98.5
TRF(20)	10.8	21.3	55.9	86.9	98.1
TRF(25)	11.1	21.4	54.3	85.7	97.8
SWF(2.1)	9.5	21.6	55.5	87.3	98.4
KW	10.4	22.3	58.1	88.7	98.7

TABLE 11-(4)

SAMPLE SIZES= 20 20 20 20 20 20
DISTRIBUTION: 10% 3N

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.9	3.4	21.4	59.5	90.0
	TRF(5)	1.0	3.9	27.8	72.1	95.9
	TRF(10)	1.0	4.1	30.2	75.6	97.3
	TRF(15)	1.1	4.2	29.6	74.6	97.3
	TRF(20)	1.0	3.8	27.9	72.6	97.0
	TRF(25)	1.3	3.5	25.9	69.0	95.8
	SWF(2.1)	0.8	3.4	27.4	72.3	96.8
	KW	0.8	3.8	28.6	74.4	97.2
5						
	TRF(0)	5.2	12.4	43.8	79.4	96.8
	TRF(5)	5.0	13.8	53.0	88.4	99.2
	TRF(10)	5.0	14.0	55.1	90.7	99.5
	TRF(15)	5.1	14.0	54.6	90.8	99.5
	TRF(20)	5.2	14.1	52.7	89.8	99.3
	TRF(25)	5.6	13.8	50.4	87.9	99.2
	SWF(2.1)	4.5	13.3	53.0	90.0	99.5
	KW	4.9	14.3	55.1	90.8	99.5
10						
	TRF(0)	10.0	20.8	56.8	87.2	98.5
	TRF(5)	9.6	22.9	65.7	94.0	99.6
	TRF(10)	9.9	23.7	67.4	95.2	99.9
	TRF(15)	9.9	23.4	67.3	94.8	99.8
	TRF(20)	10.1	23.1	66.1	94.4	99.7
	TRF(25)	10.2	23.1	63.8	93.3	99.7
	SWF(2.1)	9.5	22.9	65.9	94.7	99.8
	KW	9.8	23.5	68.4	95.2	99.9

TABLE 11-(5)

SAMPLE SIZES= 10 10 15 15 20 20
DISTRIBUTION: 10% 3N

ALPHA		PHI 0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.9	3.1	20.3	58.7	88.8
	TRF(5)	0.9	4.1	26.5	69.6	95.0
	TRF(10)	1.0	4.3	27.8	73.3	96.5
	TRF(15)	1.1	4.3	27.3	72.8	96.7
	TRF(20)	1.3	4.3	25.3	69.7	95.5
	TRF(25)	1.3	4.4	24.6	66.6	93.6
	SWF(1.8)	1.0	3.4	23.1	67.5	94.8
	SWF(2.1)	0.8	3.6	24.4	69.4	95.5
	SWF(2.4)	0.8	3.7	25.4	70.7	96.0
	KW	0.7	3.4	26.4	71.7	96.4
5						
	TRF(0)	4.9	12.3	42.6	79.4	96.4
	TRF(5)	5.1	13.9	49.9	87.3	98.8
	TRF(10)	5.3	14.4	51.1	89.7	99.4
	TRF(15)	5.3	14.3	51.4	89.6	99.4
	TRF(20)	5.3	14.4	49.7	87.9	99.0
	TRF(25)	5.5	14.6	48.0	86.3	98.8
	SWF(1.8)	4.7	13.1	47.4	87.1	99.0
	SWF(2.1)	4.7	13.1	48.8	87.7	99.2
	SWF(2.4)	5.1	13.4	49.7	88.1	99.2
	KW	5.0	14.0	51.5	89.7	99.4
10						
	TRF(0)	10.3	20.6	55.3	87.4	98.3
	TRF(5)	10.8	23.3	62.3	93.0	99.6
	TRF(10)	10.0	23.6	65.0	94.4	99.7
	TRF(15)	10.3	24.1	65.0	94.4	99.7
	TRF(20)	10.3	23.4	63.2	93.2	99.7
	TRF(25)	10.8	23.1	61.9	92.2	99.5
	SWF(1.8)	9.9	22.1	61.3	93.2	99.7
	SWF(2.1)	10.0	22.3	62.7	93.6	99.7
	SWF(2.4)	10.2	22.4	63.3	93.9	99.7
	KW	9.8	23.5	64.9	94.8	99.7

TABLE 12

Powers of some statistics for equal variances under 10% 10N

Legend

TRF(100g): Trimmed F, $F_t(g)$

SWF(k) : Sine-wave F, $F_s(k)$

KW : Kruskal-Wallis

TABLE 12-(1)

SAMPLE SIZES= 10 10 10 10

DISTRIBUTION: 10% 10N

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.1	6.2	29.8	55.7	74.6
	TRF(5)	0.1	8.3	45.6	75.7	91.2
	TRF(10)	0.5	21.9	75.8	89.9	95.8
	TRF(15)	0.5	22.7	81.6	95.1	98.9
	TRF(20)	1.0	25.1	89.2	98.2	99.3
	TRF(25)	1.0	24.1	88.3	98.8	99.7
	SWF(2.1)	0.6	25.7	92.7	99.4	99.9
	KW	0.7	24.6	84.3	97.3	99.4
5						
	TRF(0)	2.8	17.0	47.5	72.0	87.2
	TRF(5)	2.2	23.1	65.4	87.5	96.5
	TRF(10)	3.4	43.5	84.6	94.4	98.2
	TRF(15)	3.5	46.1	90.6	97.9	99.6
	TRF(20)	4.6	50.0	96.3	99.1	99.7
	TRF(25)	5.0	49.3	96.8	99.6	99.9
	SWF(2.1)	4.0	53.6	98.1	99.8	100.0
	KW	4.7	50.7	95.6	99.6	99.9
10						
	TRF(0)	7.5	27.2	57.5	79.6	91.6
	TRF(5)	6.6	34.4	74.5	92.0	98.1
	TRF(10)	8.1	55.8	89.0	96.1	99.0
	TRF(15)	8.1	59.0	94.0	98.9	99.8
	TRF(20)	9.7	63.7	97.7	99.4	99.9
	TRF(25)	10.3	63.6	98.3	99.8	99.9
	SWF(2.1)	8.8	68.0	99.0	99.9	100.0
	KW	10.1	65.1	98.2	99.9	99.9

TABLE 12-(2)

SAMPLE SIZES= 20 20 20 20
DISTRIBUTION: 10% 1ON

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.4	3.5	24.8	52.5	77.1
	TRF(5)	0.6	14.3	62.3	86.7	96.2
	TRF(10)	0.7	27.3	87.1	96.7	99.5
	TRF(15)	0.8	31.8	95.4	99.4	99.9
	TRF(20)	1.0	31.7	96.7	99.9	100.0
	TRF(25)	1.1	30.0	95.8	100.0	100.0
	SWF(1.8)	0.8	34.5	98.4	100.0	100.0
	SWF(2.1)	0.8	35.4	98.4	100.0	100.0
	SWF(2.4)	0.8	36.0	98.3	100.0	100.0
	KW	0.9	30.1	95.1	99.9	100.0
5						
	TRF(0)	3.3	12.8	42.9	71.8	89.1
	TRF(5)	2.7	30.5	76.3	93.6	98.6
	TRF(10)	4.0	49.5	93.8	98.4	99.8
	TRF(15)	4.8	55.4	98.4	99.8	99.9
	TRF(20)	5.1	56.0	99.1	100.0	100.0
	TRF(25)	5.7	54.7	99.0	100.0	100.0
	SWF(1.8)	4.4	60.9	99.7	100.0	100.0
	SWF(2.1)	4.3	61.7	99.7	100.0	100.0
	SWF(2.4)	4.5	61.9	99.7	100.0	100.0
	KW	4.8	55.4	98.7	100.0	100.0
10						
	TRF(0)	8.2	21.3	54.7	80.2	93.6
	TRF(5)	6.5	42.6	82.6	95.9	99.3
	TRF(10)	7.9	62.0	95.9	99.2	99.9
	TRF(15)	9.3	67.9	99.0	99.9	100.0
	TRF(20)	9.9	68.5	99.6	100.0	100.0
	TRF(25)	10.8	67.5	99.6	100.0	100.0
	SWF(1.8)	9.2	74.0	99.9	100.0	100.0
	SWF(2.1)	8.9	74.1	99.9	100.0	100.0
	SWF(2.4)	8.9	73.8	99.8	100.0	100.0
	KW	10.1	67.5	99.4	100.0	100.0

TABLE 12-(3)

SAMPLE SIZES= 10 15 15 20
DISTRIBUTION: 10% 1ON

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.4	4.6	25.7	54.2	76.5
	TRF(5)	0.5	12.4	58.2	83.8	95.1
	TRF(10)	0.7	22.3	82.7	95.4	98.6
	TRF(15)	1.0	27.2	91.8	98.4	99.6
	TRF(20)	1.1	28.5	95.1	99.4	99.8
	TRF(25)	1.3	27.4	94.5	99.7	99.9
	SWF(2.1)	0.8	30.8	96.8	99.8	100.0
	KW	0.8	27.0	92.5	99.6	99.9
5						
	TRF(0)	3.3	14.6	44.4	71.7	89.2
	TRF(5)	3.2	28.5	74.1	91.3	98.0
	TRF(10)	3.5	45.1	91.2	97.8	99.4
	TRF(15)	4.2	53.1	96.3	99.3	99.9
	TRF(20)	4.7	54.8	98.5	99.6	100.0
	TRF(25)	5.5	52.5	98.9	99.9	100.0
	SWF(2.1)	4.3	58.8	99.2	99.9	100.0
	KW	4.5	53.5	98.4	99.9	100.0
10						
	TRF(0)	8.1	23.6	56.1	79.5	93.0
	TRF(5)	8.2	40.2	81.4	94.5	98.8
	TRF(10)	7.6	57.3	94.0	98.5	99.7
	TRF(15)	8.7	65.5	97.9	99.6	100.0
	TRF(20)	9.8	67.4	99.3	99.7	100.0
	TRF(25)	10.1	66.8	99.6	99.9	100.0
	SWF(2.1)	9.1	71.8	99.6	99.9	100.0
	KW	10.1	67.6	99.2	100.0	100.0

TABLE 12-(4)

SAMPLE SIZES= 20 20 20 20 20 20
DISTRIBUTION: 10% 1 ON

ALPHA	PHI 0.0	0.5	1.0	1.5	2.0
1					
TRF(0)	0.5	3.5	26.3	62.7	87.7
TRF(5)	0.4	16.0	68.2	92.8	98.8
TRF(10)	0.8	33.9	92.1	98.8	99.9
TRF(15)	1.0	41.7	98.2	99.7	100.0
TRF(20)	1.1	42.7	99.4	100.0	100.0
TRF(25)	1.3	40.2	99.5	100.0	100.0
SWF(2.1)	1.0	48.4	99.9	100.0	100.0
KW	0.9	40.9	99.0	100.0	100.0
5					
TRF(0)	3.9	13.4	48.1	80.9	95.4
TRF(5)	2.5	33.4	81.9	97.2	99.6
TRF(10)	3.8	57.2	96.5	99.6	100.0
TRF(15)	4.9	66.1	99.3	99.9	100.0
TRF(20)	5.1	67.3	99.8	100.0	100.0
TRF(25)	5.4	65.4	99.9	100.0	100.0
SWF(2.1)	4.7	73.5	100.0	100.0	100.0
KW	5.0	66.2	99.9	100.0	100.0
10					
TRF(0)	8.4	22.9	59.4	87.2	97.3
TRF(5)	6.1	43.9	87.8	98.4	99.9
TRF(10)	7.7	67.7	97.8	99.8	100.0
TRF(15)	9.8	76.9	99.6	99.9	100.0
TRF(20)	10.5	77.8	99.9	100.0	100.0
TRF(25)	10.6	76.5	99.9	100.0	100.0
SWF(2.1)	9.2	82.7	100.0	100.0	100.0
KW	10.2	77.7	100.0	100.0	100.0

TABLE 12-(5)

SAMPLE SIZES= 10 10 15 15 20 20
DISTRIBUTION: 10% 1ON

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.5	4.7	28.8	62.7	86.4
	TRF(5)	0.7	14.4	63.8	90.3	97.9
	TRF(10)	0.8	27.7	88.4	97.9	99.7
	TRF(15)	0.9	35.5	95.2	99.5	99.9
	TRF(20)	1.1	38.6	98.1	99.6	99.9
	TRF(25)	1.2	37.7	98.7	99.9	100.0
	SWF(1.8)	0.7	41.8	99.4	100.0	100.0
	SWF(2.1)	0.7	43.0	99.3	100.0	100.0
	SWF(2.4)	0.8	43.3	99.1	100.0	100.0
	KW	0.8	36.6	98.2	100.0	100.0
5						
	TRF(0)	3.7	14.4	49.3	80.4	94.1
	TRF(5)	4.8	30.9	79.7	96.1	99.4
	TRF(10)	4.0	50.0	94.3	99.1	100.0
	TRF(15)	4.6	60.2	97.9	99.8	100.0
	TRF(20)	5.0	64.0	99.4	99.9	100.0
	TRF(25)	5.6	63.2	99.6	99.9	100.0
	SWF(1.8)	4.2	68.7	99.8	100.0	100.0
	SWF(2.1)	4.3	69.2	99.9	100.0	100.0
	SWF(2.4)	4.3	68.5	99.8	100.0	100.0
	KW	4.7	64.1	99.7	100.0	100.0
10						
	TRF(0)	8.3	24.0	59.8	86.7	96.8
	TRF(5)	9.2	43.1	86.2	97.5	99.8
	TRF(10)	8.2	62.4	96.3	99.6	100.0
	TRF(15)	9.1	72.3	98.9	99.9	100.0
	TRF(20)	10.0	76.1	99.6	99.9	100.0
	TRF(25)	10.7	74.8	99.8	100.0	100.0
	SWF(1.8)	9.4	79.7	100.0	100.0	100.0
	SWF(2.1)	9.3	79.9	100.0	100.0	100.0
	SWF(2.4)	9.2	79.5	99.9	100.0	100.0
	KW	9.4	76.0	99.9	100.0	100.0

TABLE 13

Powers of some statistics for equal variances under D-EXP

Legend

TRF(100g): Trimmed F, $F_t(g)$
SWF(k) : Sine-wave F, $F_s(k)$
KW : Kruskal-Wallis

TABLE 13-(1)

SAMPLE SIZES= 10 10 10 10
DISTRIBUTION: D-EXP

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.5	2.2	15.2	42.8	74.2
	TRF(5)	0.5	2.3	16.[illegible]	47.5	80.2
	TRF(10)	0.7	2.8	18.0	49.6	81.0
	TRF(15)	0.8	3.0	19.1	52.5	83.4
	TRF(20)	1.0	3.2	19.4	51.9	82.0
	TRF(25)	1.0	3.2	19.0	50.8	81.5
	SWF(2.1)	0.6	2.1	13.7	43.1	76.8
	KW	0.6	2.6	17.1	47.9	78.9
5						
	TRF(0)	4.1	10.9	34.6	68.3	89.8
	TRF(5)	3.9	11.6	37.6	73.2	93.0
	TRF(10)	4.0	12.0	39.1	74.6	94.1
	TRF(15)	4.1	12.0	41.5	77.6	95.1
	TRF(20)	4.4	12.3	41.8	76.7	94.4
	TRF(25)	4.5	12.2	41.8	77.4	94.7
	SWF(2.1)	3.5	9.7	35.4	71.7	92.9
	KW	4.2	12.8	41.4	75.8	93.7
10						
	TRF(0)	9.7	19.3	47.6	78.8	94.5
	TRF(5)	9.3	20.3	50.9	83.1	96.2
	TRF(10)	9.0	20.4	52.7	84.6	97.0
	TRF(15)	9.1	21.1	54.5	86.5	97.5
	TRF(20)	9.1	21.5	54.9	86.0	97.4
	TRF(25)	9.0	21.6	55.9	86.4	97.6
	SWF(2.1)	7.9	17.5	49.4	83.0	96.4
	KW	9.5	22.3	55.2	85.8	97.0

TABLE 13-(2)

SAMPLE SIZES= 20 20 20 20
DISTRIBUTION: D-EXP

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	1.0	3.0	16.4	46.5	78.1
	TRF(5)	1.1	3.4	19.5	53.0	84.2
	TRF(10)	0.9	3.8	21.0	57.7	87.5
	TRF(15)	0.8	4.2	23.2	60.9	89.4
	TRF(20)	1.0	4.3	24.5	63.2	90.4
	TRF(25)	1.1	4.5	25.5	63.6	90.7
	SWF(1.8)	0.7	3.0	18.4	54.9	86.1
	SWF(2.1)	0.7	3.2	18.7	55.2	86.8
	SWF(2.4)	0.8	3.1	19.2	56.0	86.8
	KW	0.9	3.9	23.6	59.4	87.9
5						
	TRF(0)	4.9	11.5	35.6	69.5	91.3
	TRF(5)	4.9	11.9	40.3	75.1	94.6
	TRF(10)	4.7	12.8	43.9	79.3	96.1
	TRF(15)	4.8	13.5	46.4	81.4	97.0
	TRF(20)	4.7	13.7	48.5	83.0	97.2
	TRF(25)	5.0	14.2	49.4	83.5	97.3
	SWF(1.8)	4.0	10.9	42.2	78.9	96.1
	SWF(2.1)	4.0	11.4	42.4	78.8	96.1
	SWF(2.4)	4.2	11.5	42.6	78.7	96.2
	KW	5.2	13.5	47.2	81.3	96.5
10						
	TRF(0)	10.1	19.6	48.8	79.7	95.4
	TRF(5)	9.8	20.7	53.7	84.6	97.2
	TRF(10)	9.8	21.6	57.3	87.5	98.1
	TRF(15)	9.7	22.1	59.0	88.9	98.5
	TRF(20)	10.2	22.6	61.0	90.0	98.7
	TRF(25)	9.9	22.9	62.5	90.1	98.8
	SWF(1.8)	8.6	19.8	56.5	87.1	98.2
	SWF(2.1)	9.2	20.5	56.4	87.4	98.2
	SWF(2.4)	9.2	20.5	56.3	86.7	98.2
	KW	10.4	23.2	60.5	89.0	98.4

TABLE 13-(3)

SAMPLE SIZES= 10 15 15 20
DISTRIBUTION:D-EXP

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.8	3.3	14.7	44.4	77.3
	TRF(5)	0.9	3.3	17.8	50.0	83.4
	TRF(10)	0.9	3.6	19.8	53.3	86.4
	TRF(15)	0.8	3.8	21.0	56.8	87.9
	TRF(20)	0.9	4.3	21.8	57.8	88.2
	TRF(25)	0.9	4.3	22.3	57.5	88.0
	SWF(2.1)	0.7	3.0	16.4	49.3	83.2
	KW	0.7	3.5	20.0	53.4	86.1
5						
	TRF(0)	4.7	10.6	34.6	68.5	91.5
	TRF(5)	4.7	11.5	38.9	74.1	94.6
	TRF(10)	4.8	12.5	41.9	77.4	95.7
	TRF(15)	4.8	13.4	43.9	79.4	96.3
	TRF(20)	5.0	13.3	45.0	80.2	96.5
	TRF(25)	5.2	13.6	46.3	80.9	96.5
	SWF(2.1)	3.9	10.9	39.1	75.3	95.3
	KW	4.4	13.1	43.7	78.7	96.2
10						
	TRF(0)	9.8	18.7	48.0	78.6	95.5
	TRF(5)	9.7	19.8	52.4	83.7	97.4
	TRF(10)	9.4	20.7	54.3	86.1	98.0
	TRF(15)	9.6	21.3	56.6	87.5	98.3
	TRF(20)	9.8	22.0	57.9	87.8	98.2
	TRF(25)	9.7	22.6	59.3	88.9	98.3
	SWF(2.1)	8.4	19.1	52.6	84.6	97.9
	KW	9.7	22.6	57.8	87.4	98.3

5

TABLE 13-(4)

SAMPLE SIZES= 20 20 20 20 20 20

DISTRIBUTION: D-EXP

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.7	3.6	20.6	58.4	89.8
	TRF(5)	0.9	3.9	24.6	66.9	94.3
	TRF(10)	1.0	4.6	27.8	71.9	96.3
	TRF(15)	1.1	4.6	29.7	75.6	97.0
	TRF(20)	1.3	4.5	31.8	77.4	97.5
	TRF(25)	1.2	4.5	32.6	78.4	97.6
	SWF(2.1)	0.9	3.6	24.9	70.5	95.8
	KW	0.9	4.8	30.4	75.0	96.6
5						
	TRF(0)	5.0	13.1	42.2	80.0	96.8
	TRF(5)	5.1	13.1	47.5	85.9	98.5
	TRF(10)	5.0	13.9	50.7	89.1	99.1
	TRF(15)	4.9	15.0	53.8	90.4	99.3
	TRF(20)	5.5	16.3	56.0	91.2	99.4
	TRF(25)	5.3	16.0	57.5	91.7	99.5
	SWF(2.1)	4.5	12.7	50.3	88.4	99.0
	KW	5.0	15.6	55.9	90.7	99.3
10						
	TRF(0)	9.6	21.5	55.0	87.3	98.5
	TRF(5)	9.5	22.4	60.1	91.7	99.5
	TRF(10)	9.9	23.6	63.5	93.8	99.6
	TRF(15)	9.7	24.9	66.9	94.9	99.7
	TRF(20)	10.2	25.7	69.1	95.3	99.7
	TRF(25)	9.7	26.3	70.0	95.7	99.7
	SWF(2.1)	9.1	22.1	63.2	93.4	99.5
	KW	10.0	25.5	68.1	95.2	99.7

TABLE 13-(5)

SAMPLE SIZES= 10 10 15 15 20 20
DISTRIBUTION: D-EXP

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	1.0	3.2	20.1	59.0	90.1
	TRF(5)	0.8	3.9	24.2	66.6	94.1
	TRF(10)	0.9	4.3	26.8	70.5	95.5
	TRF(15)	0.8	4.8	28.1	73.4	96.3
	TRF(20)	0.9	4.7	29.6	74.1	96.5
	TRF(25)	0.9	4.9	29.9	74.5	96.7
	SWF(1.8)	0.6	2.9	22.5	65.5	94.2
	SWF(2.1)	0.5	3.3	23.2	67.1	94.6
	SWF(2.4)	0.6	3.5	24.1	67.8	95.2
	KW	0.6	4.1	27.5	70.7	95.0
5						
	TRF(0)	4.9	12.4	41.7	79.2	96.6
	TRF(5)	4.6	13.7	46.6	84.7	98.4
	TRF(10)	4.6	14.5	50.2	87.2	98.9
	TRF(15)	4.8	15.1	53.4	89.0	99.2
	TRF(20)	5.0	15.5	53.8	89.7	99.3
	TRF(25)	4.9	16.2	55.4	90.5	99.2
	SWF(1.8)	3.5	12.2	46.5	85.8	99.0
	SWF(2.1)	3.5	13.0	47.6	86.6	99.1
	SWF(2.4)	3.9	13.5	47.4	86.8	99.0
	KW	4.6	15.6	53.0	88.9	99.1
10						
	TRF(0)	9.4	21.4	54.2	86.9	98.5
	TRF(5)	9.2	22.9	60.0	90.9	99.3
	TRF(10)	9.0	23.5	63.2	92.6	99.4
	TRF(15)	9.8	24.2	66.0	93.9	99.6
	TRF(20)	10.0	25.1	66.7	94.5	99.7
	TRF(25)	10.5	25.6	68.3	95.0	99.8
	SWF(1.8)	8.1	20.6	60.7	92.6	99.5
	SWF(2.1)	8.4	21.2	61.5	92.6	99.6
	SWF(2.4)	8.5	21.7	61.9	92.6	99.6
	KW	9.6	25.3	67.1	94.0	99.7

TABLE 14

Powers of some statistics for equal variances under 25% 1/U

Legend
TRF(100g): Trimmed F, $F_t(g)$
SWF(k) : Sine-wave F, $F_S(k)$
KW : Kruskal-Wallis

TABLE 14-(1)

SAMPLE SIZES= 10 10 10 10
DISTRIBUTION: 25% 1/U

ALPHA		PHI 0.0	0.5	1.0	1.5	2.0
.1						
	TRF(0)	0.2	0.7	4.1	11.3	24.3
	TRF(5)	0.3	0.9	5.0	14.5	31.2
	TRF(10)	0.7	1.8	9.1	27.5	55.6
	TRF(15)	0.6	1.9	9.3	28.1	57.0
	TRF(20)	1.1	2.4	9.9	28.2	57.6
	TRF(25)	0.9	2.5	9.6	26.8	55.3
	SWF(2.1)	0.7	2.1	8.4	26.7	56.5
	KW	0.5	1.7	9.1	27.9	56.2
5						
	TRF(0)	2.5	3.8	12.4	25.0	39.0
	TRF(5)	2.3	4.6	14.7	30.7	47.9
	TRF(10)	4.1	8.3	25.1	51.0	76.9
	TRF(15)	4.1	8.7	26.3	53.2	79.3
	TRF(20)	4.7	10.0	26.8	55.0	81.1
	TRF(25)	4.8	9.8	27.1	54.4	80.3
	SWF(2.1)	3.7	8.3	26.0	54.0	81.5
	KW	4.4	9.4	26.7	55.4	81.5
10						
	TRF(0)	6.1	8.8	19.7	33.9	47.8
	TRF(5)	5.9	9.4	23.8	41.7	56.9
	TRF(10)	8.3	14.7	37.7	64.0	84.7
	TRF(15)	8.6	15.8	38.9	66.3	87.7
	TRF(20)	9.2	17.0	40.0	67.8	89.3
	TRF(25)	9.4	17.4	40.0	68.1	89.1
	SWF(2.1)	8.6	15.3	38.6	68.7	90.2
	KW	9.8	17.2	40.5	69.3	89.5

TABLE 14-(2)

SAMPLE SIZES= 20 20 20 20
DISTRIBUTION: 25% 1/U

ALPHA	PHI 0.0	0.5	1.0	1.5	2.0
1					
TRF(0)	0.3	0.8	1.7	6.9	14.4
TRF(5)	0.8	1.6	7.5	25.4	50.3
TRF(10)	0.9	2.0	10.7	34.2	65.9
TRF(15)	0.9	2.4	11.7	37.3	69.4
TRF(20)	1.1	2.6	11.8	37.0	68.2
TRF(25)	1.1	2.6	11.1	34.4	65.7
SWF(1.8)	0.8	2.1	10.8	35.7	69.3
SWF(2.1)	0.8	2.1	11.5	36.7	70.3
SWF(2.4)	0.8	2.2	11.8	37.1	70.9
KW	1.0	2.1	11.3	34.9	66.2
5					
TRF(0)	2.1	3.5	7.2	16.9	26.9
TRF(5)	3.4	7.6	21.2	46.2	69.5
TRF(10)	4.7	9.4	28.1	59.2	84.7
TRF(15)	5.1	9.5	29.9	62.3	87.4
TRF(20)	5.2	9.5	30.0	61.4	87.3
TRF(25)	5.2	9.9	28.8	60.2	86.0
SWF(1.8)	4.1	9.1	29.7	62.4	88.9
SWF(2.1)	4.2	9.1	30.6	63.3	89.2
SWF(2.4)	4.4	9.2	30.4	63.6	88.6
KW	5.0	9.8	29.0	61.7	86.5
10					
TRF(0)	5.5	7.9	13.3	25.2	35.4
TRF(5)	7.7	13.5	32.8	58.4	78.6
TRF(10)	9.3	16.4	40.5	70.9	90.6
TRF(15)	10.0	17.3	42.4	73.9	93.1
TRF(20)	10.3	17.3	42.3	73.7	93.1
TRF(25)	10.8	17.5	41.8	72.3	92.2
SWF(1.8)	9.4	16.7	42.7	75.3	94.2
SWF(2.1)	9.4	16.7	43.2	76.0	94.2
SWF(2.4)	9.2	16.9	42.9	75.9	94.1
KW	10.0	17.3	42.4	73.5	93.0

TABLE 14-(3)

SAMPLE SIZES= 10 15 15 20
DISTRIBUTION: 25% 1/U

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.3	0.9	3.1	9.1	18.6
	TRF(5)	0.7	1.7	6.1	19.2	37.5
	TRF(10)	0.9	2.3	10.3	31.7	60.5
	TRF(15)	1.0	2.6	11.4	34.2	65.5
	TRF(20)	1.1	2.9	11.2	34.4	65.3
	TRF(25)	1.1	2.8	11.3	32.6	62.2
	SWF(2.1)	0.7	2.3	10.5	33.7	65.6
	KW	0.8	2.2	10.3	32.6	62.6
5						
	TRF(0)	2.4	3.8	9.7	21.0	31.9
	TRF(5)	3.4	6.2	17.5	37.8	56.1
	TRF(10)	4.3	8.4	26.1	56.3	79.5
	TRF(15)	4.9	9.8	29.1	60.6	84.3
	TRF(20)	4.9	10.1	29.1	60.0	84.5
	TRF(25)	5.3	10.2	28.4	58.9	83.4
	SWF(2.1)	4.3	9.5	29.3	61.2	86.0
	KW	4.7	9.2	28.7	60.3	83.8
10						
	TRF(0)	5.9	8.4	17.0	29.6	40.6
	TRF(5)	8.2	11.8	27.0	49.8	65.6
	TRF(10)	8.6	15.2	38.0	67.8	86.9
	TRF(15)	9.8	16.3	41.4	72.1	90.4
	TRF(20)	10.3	17.4	41.3	72.8	91.0
	TRF(25)	10.3	17.4	41.2	71.8	90.4
	SWF(2.1)	8.7	16.4	41.5	73.3	92.3
	KW	9.4	16.7	41.7	72.2	90.7

TABLE 14-(4)

SAMPLE SIZES= 20 20 20 20 20 20
DISTRIBUTION: 25% 1/U

ALPHA		PHI 0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.2	0.3	1.7	5.5	12.8
	TRF(5)	0.4	1.6	8.8	29.8	59.1
	TRF(10)	0.8	2.8	14.0	44.9	80.0
	TRF(15)	0.9	3.2	15.4	48.4	84.1
	TRF(20)	1.0	3.2	15.8	48.0	83.9
	TRF(25)	0.9	3.3	15.0	45.3	81.8
	SWF(2.1)	0.6	2.6	14.9	49.8	85.8
	KW	0.8	2.9	14.5	47.4	82.7
5						
	TRF(0)	1.8	2.5	6.7	13.9	24.1
	TRF(5)	2.9	6.9	23.7	51.4	75.5
	TRF(10)	3.9	9.4	32.9	68.4	91.9
	TRF(15)	4.4	10.0	35.8	72.6	94.7
	TRF(20)	4.6	10.5	35.9	71.9	94.7
	TRF(25)	4.6	10.5	35.0	71.0	94.0
	SWF(2.1)	3.8	10.0	36.1	74.1	96.3
	KW	4.3	10.3	35.7	71.9	94.5
10						
	TRF(0)	4.7	5.7	12.3	21.9	31.8
	TRF(5)	6.6	12.9	33.9	62.2	81.9
	TRF(10)	8.5	16.4	45.4	78.5	95.2
	TRF(15)	9.4	17.8	49.1	82.6	97.5
	TRF(20)	9.3	18.3	48.8	82.6	97.5
	TRF(25)	9.3	17.9	47.8	81.4	97.5
	SWF(2.1)	8.7	17.1	49.5	84.6	98.3
	KW	9.5	17.7	48.8	82.7	97.4

TABLE 14-(5)

SAMPLE SIZES= 10 10 15 15 20 20
DISTRIBUTION: 25% 1/U

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.5	0.7	2.4	8.0	16.3
	TRF(5)	0.6	1.4	6.8	23.0	42.6
	TRF(10)	0.9	2.2	12.8	40.6	72.9
	TRF(15)	0.9	2.4	14.6	44.8	78.1
	TRF(20)	1.2	2.4	14.8	45.5	79.3
	TRF(25)	1.1	2.4	14.4	43.3	76.9
	SWF(1.8)	0.8	2.2	13.5	44.9	79.9
	SWF(2.1)	0.7	2.2	14.0	46.6	80.7
	SWF(2.4)	0.8	2.4	14.5	47.2	80.8
	KW	0.7	2.3	13.9	43.9	78.2
5						
	TRF(0)	2.7	3.6	9.0	18.1	28.6
	TRF(5)	4.2	7.0	19.8	40.8	60.8
	TRF(10)	4.4	9.6	30.6	63.9	86.6
	TRF(15)	5.2	10.4	33.6	69.3	91.1
	TRF(20)	5.6	10.9	34.0	70.0	92.6
	TRF(25)	5.9	11.1	33.8	68.9	91.8
	SWF(1.8)	4.4	9.7	33.1	71.1	93.3
	SWF(2.1)	4.4	10.0	33.6	72.3	93.9
	SWF(2.4)	4.4	10.3	34.2	72.2	93.8
	KW	4.5	10.4	34.1	69.7	92.4
10						
	TRF(0)	6.4	8.3	16.1	26.5	37.7
	TRF(5)	9.6	14.5	30.3	51.9	69.2
	TRF(10)	9.1	17.1	42.1	74.6	91.6
	TRF(15)	10.5	18.8	45.6	79.6	95.4
	TRF(20)	10.5	19.1	46.2	80.2	96.5
	TRF(25)	11.3	19.1	45.5	79.0	96.3
	SWF(1.8)	9.6	18.2	45.7	81.8	97.1
	SWF(2.1)	9.5	18.3	46.6	82.4	97.1
	SWF(2.4)	9.8	18.6	47.2	82.2	96.9
	KW	9.5	19.2	47.2	80.1	96.4

TABLE 15

Powers of some statistics for equal variances under 25% 3/U

Legend

TRF(100g): Trimmed F, $F_t(g)$

SWF(k) : Sine-wave F, $F_S(k)$

KW : Kruskal-Wallis

TABLE 15-(1)

SAMPLE SIZES= 10 10 10 10

DISTRIBUTION: 25% 3/U

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.1	0.2	0.9	2.5	5.0
	TRF(5)	0.1	0.2	1.0	3.2	7.8
	TRF(10)	0.3	1.1	4.4	14.9	29.0
	TRF(15)	0.3	0.9	4.9	17.1	34.9
	TRF(20)	0.6	1.7	8.8	25.7	50.3
	TRF(25)	0.6	1.7	8.6	25.2	50.8
	SWF(2.1)	0.5	1.5	8.7	26.2	52.8
	KW	0.6	2.2	8.6	22.6	43.3
5						
	TRF(0)	1.2	1.8	3.9	8.2	12.3
	TRF(5)	1.0	1.6	4.7	10.6	17.7
	TRF(10)	2.2	4.7	14.5	30.9	47.4
	TRF(15)	2.1	4.9	17.2	36.7	56.0
	TRF(20)	3.3	7.6	24.1	50.2	71.5
	TRF(25)	4.1	8.0	25.1	51.4	73.8
	SWF(2.1)	3.3	7.7	25.6	53.0	74.7
	KW	4.3	9.1	26.5	49.8	70.3
10						
	TRF(0)	4.0	5.0	8.3	13.2	18.7
	TRF(5)	3.2	4.7	9.6	16.9	26.2
	TRF(10)	5.4	10.0	23.1	41.9	57.3
	TRF(15)	5.5	10.4	26.6	49.0	66.3
	TRF(20)	8.0	14.0	35.4	62.2	79.8
	TRF(25)	8.6	14.6	36.8	64.8	82.3
	SWF(2.1)	8.0	14.4	37.4	65.3	83.6
	KW	9.4	16.8	39.1	63.1	81.0

TABLE 15-(2)

SAMPLE SIZES= 20 20 20 20
DISTRIBUTION: 25% 3/U

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.1	0.1	0.5	0.8	1.8
	TRF(5)	0.4	0.5	1.9	5.7	13.5
	TRF(10)	0.5	1.1	4.8	16.7	35.1
	TRF(15)	0.6	1.7	8.9	28.7	54.5
	TRF(20)	0.8	1.9	11.6	35.3	64.3
	TRF(25)	1.0	2.5	12.6	37.1	67.3
	SWF(1.8)	0.8	1.9	12.8	40.5	72.7
	SWF(2.1)	0.8	1.6	12.6	38.6	69.9
	SWF(2.4)	0.8	1.7	12.3	37.2	67.4
	KW	0.8	2.6	10.9	30.3	56.3
5						
	TRF(0)	1.7	1.9	3.0	4.4	7.3
	TRF(5)	2.0	3.4	7.6	17.3	28.2
	TRF(10)	2.5	5.2	15.8	34.9	55.1
	TRF(15)	3.8	7.2	23.8	51.5	73.8
	TRF(20)	4.2	8.6	28.8	58.8	83.3
	TRF(25)	4.6	9.5	30.5	62.3	85.9
	SWF(1.8)	3.8	9.6	32.1	65.4	88.9
	SWF(2.1)	4.2	9.3	31.3	63.4	86.9
	SWF(2.4)	4.6	9.1	30.1	61.1	84.7
	KW	5.0	9.6	27.9	56.4	79.1
10						
	TRF(0)	4.6	5.6	6.7	9.6	13.0
	TRF(5)	5.1	7.3	13.7	26.1	38.7
	TRF(10)	6.1	10.9	26.0	46.7	65.6
	TRF(15)	7.4	13.8	36.0	63.0	82.3
	TRF(20)	8.5	16.2	41.7	71.0	89.6
	TRF(25)	9.6	17.4	43.2	74.0	92.1
	SWF(1.8)	8.3	17.5	45.4	76.7	94.0
	SWF(2.1)	8.5	17.3	44.0	75.4	92.5
	SWF(2.4)	8.8	16.8	43.1	73.1	91.1
	KW	10.1	16.9	39.9	68.6	87.2

TABLE 15-(3)

SAMPLE SIZES= 10 15 15 20
DISTRIBUTION: 25% 3/U

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.3	0.3	0.8	1.2	3.0
	TRF(5)	0.3	0.4	1.5	3.9	10.3
	TRF(10)	0.4	0.8	4.7	14.5	31.2
	TRF(15)	0.5	1.5	7.9	24.7	47.9
	TRF(20)	0.5	2.2	10.7	31.1	60.2
	TRF(25)	0.8	2.5	11.7	32.1	62.1
	SWF(2.1)	0.6	2.2	11.7	33.8	64.1
	KW	0.7	2.4	9.6	27.7	52.8
5						
	TRF(0)	1.9	2.2	3.9	5.8	9.6
	TRF(5)	2.3	3.1	7.2	13.2	22.8
	TRF(10)	1.9	5.2	14.3	31.4	50.9
	TRF(15)	3.2	6.9	21.9	46.4	68.0
	TRF(20)	3.9	8.7	27.1	55.6	78.9
	TRF(25)	4.0	9.7	28.6	58.5	81.8
	SWF(2.1)	3.9	8.9	29.1	60.4	83.5
	KW	4.6	9.6	26.9	53.3	76.8
10						
	TRF(0)	5.5	5.7	8.5	11.0	16.4
	TRF(5)	6.2	7.5	13.9	21.8	32.7
	TRF(10)	5.6	9.8	23.5	42.6	61.2
	TRF(15)	6.8	13.5	33.7	57.8	77.6
	TRF(20)	8.4	15.6	39.9	68.1	86.7
	TRF(25)	8.9	17.0	41.7	70.5	89.1
	SWF(2.1)	8.1	16.5	41.9	72.0	89.9
	KW	9.7	17.1	40.5	67.1	85.5

TABLE 15-(4)

SAMPLE SIZES= 20 20 20 20 20 20
DISTRIBUTION: 25% 3/U

ALPHA	PHI 0.0	0.5	1.0	1.5	2.0
1					
TRF(0)	0.2	0.3	0.4	0.7	1.3
TRF(5)	0.3	0.6	1.7	5.3	13.5
TRF(10)	0.4	0.9	4.8	18.5	40.9
TRF(15)	0.7	1.6	9.9	35.1	66.3
TRF(20)	0.9	2.4	13.3	45.4	78.8
TRF(25)	1.0	2.8	14.2	48.6	82.7
SWF(2.1)	0.8	2.7	15.0	49.4	83.2
KW	0.8	2.4	12.4	42.2	73.6
5					
TRF(0)	1.8	1.7	2.6	3.6	5.5
TRF(5)	2.2	2.9	6.4	15.1	28.8
TRF(10)	3.0	5.2	15.4	37.0	60.7
TRF(15)	3.5	7.8	25.5	57.4	81.7
TRF(20)	3.9	9.5	32.2	67.8	90.6
TRF(25)	4.5	11.0	34.6	71.5	93.8
SWF(2.1)	4.7	10.4	34.9	73.3	94.2
KW	5.1	10.9	32.1	65.8	89.8
10					
TRF(0)	5.0	4.5	6.8	8.0	11.3
TRF(5)	5.4	6.1	12.0	24.0	38.7
TRF(10)	6.3	10.5	24.9	48.4	70.3
TRF(15)	7.5	14.5	36.9	68.0	88.0
TRF(20)	8.1	17.3	45.0	78.2	94.4
TRF(25)	9.2	18.4	48.2	82.1	96.6
SWF(2.1)	8.9	17.9	48.1	82.9	97.0
KW	10.1	19.0	46.0	76.7	94.5

TABLE 15-(5)

SAMPLE SIZES= 10 10 15 15 20 20
DISTRIBUTION: 25% 3/U

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.4	0.4	0.6	1.3	2.4
	TRF(5)	0.3	0.6	1.8	4.2	9.6
	TRF(10)	0.7	0.9	4.9	16.4	34.6
	TRF(15)	0.7	1.6	8.8	30.0	55.3
	TRF(20)	0.8	2.7	12.3	41.6	72.0
	TRF(25)	0.9	2.9	13.3	43.9	75.5
	SWF(1.8)	0.9	2.7	14.3	46.8	79.5
	SWF(2.1)	1.2	2.7	14.0	45.5	76.7
	SWF(2.4)	1.1	2.8	14.0	43.6	74.9
	KW	0.7	2.6	12.4	38.0	67.1
5						
	TRF(0)	2.3	2.4	3.0	5.1	8.1
	TRF(5)	3.2	3.6	6.6	13.1	22.2
	TRF(10)	3.2	5.0	14.8	33.5	54.5
	TRF(15)	4.0	7.6	23.2	51.6	74.3
	TRF(20)	4.7	9.5	30.0	63.9	86.2
	TRF(25)	5.4	10.4	32.6	68.0	90.4
	SWF(1.8)	4.5	10.0	33.2	71.4	92.8
	SWF(2.1)	5.1	10.0	32.8	69.4	91.5
	SWF(2.4)	5.3	10.0	31.6	67.1	89.7
	KW	5.4	10.3	31.6	64.0	86.8
10						
	TRF(0)	5.8	6.0	6.9	10.5	14.5
	TRF(5)	7.8	9.1	13.7	22.8	32.9
	TRF(10)	7.0	10.4	23.8	44.6	64.8
	TRF(15)	8.8	14.0	34.7	62.7	81.7
	TRF(20)	9.6	16.8	43.0	74.7	91.3
	TRF(25)	10.3	18.2	45.4	78.6	94.6
	SWF(1.8)	9.2	17.4	47.2	81.5	96.0
	SWF(2.1)	9.9	17.2	45.8	79.8	95.4
	SWF(2.4)	10.4	17.3	44.2	77.7	94.3
	KW	10.4	17.6	44.6	76.0	93.3

TABLE 16

Powers of some statistics for equal variances under ALL 1/U

Legend
TRF(100g): Trimmed F, $F_t(g)$
SWF(k) : Sine-wave F, $F_S(k)$
KW : Kruskal-Wallis

TABLE 16-(1)

SAMPLE SIZES= 10 10 10 10
DISTRIBUTION: ALL 1/U

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.4	0.6	1.2	3.2	8.1
	TRF(5)	0.2	0.7	1.8	5.1	11.5
	TRF(10)	0.4	1.2	5.3	17.5	35.1
	TRF(15)	0.4	1.3	6.6	20.5	41.1
	TRF(20)	0.6	2.0	9.5	27.9	52.6
	TRF(25)	0.6	2.1	9.1	27.3	53.8
	SWF(2.1)	0.6	1.6	8.0	25.2	51.4
	KW	0.8	2.3	8.7	23.8	44.4
5						
	TRF(0)	2.0	2.9	5.5	10.7	17.5
	TRF(5)	1.8	2.8	6.7	14.5	24.3
	TRF(10)	3.2	5.9	17.7	35.7	54.7
	TRF(15)	3.0	6.3	20.2	41.4	62.8
	TRF(20)	3.8	8.4	26.0	51.6	74.1
	TRF(25)	3.9	8.5	26.3	53.5	76.9
	SWF(2.1)	3.8	7.6	24.2	51.6	74.9
	KW	4.9	10.0	25.9	50.3	70.6
10						
	TRF(0)	4.9	6.0	10.9	17.6	25.0
	TRF(5)	4.4	6.5	12.6	23.1	33.7
	TRF(10)	6.7	11.9	27.2	47.5	65.4
	TRF(15)	6.9	13.1	30.7	54.2	73.1
	TRF(20)	8.1	15.6	37.6	64.5	82.6
	TRF(25)	8.2	16.1	39.3	67.4	85.5
	SWF(2.1)	7.7	14.7	36.6	64.4	84.3
	KW	9.9	18.0	39.3	63.9	81.1

TABLE 16-(2)

SAMPLE SIZES= 20 20 20 20

DISTRIBUTION: ALL 1/U

ALPHA		PHI 0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.1	0.2	0.7	1.7	3.4
	TRF(5)	0.2	0.4	2.9	9.0	20.7
	TRF(10)	0.4	1.3	6.3	20.1	42.9
	TRF(15)	0.4	1.6	9.6	30.6	59.6
	TRF(20)	0.6	2.1	11.9	37.0	67.5
	TRF(25)	0.5	2.4	13.4	40.5	70.9
	SWF(1.8)	0.5	2.4	11.8	38.4	69.5
	SWF(2.1)	0.6	2.6	11.3	36.8	67.9
	SWF(2.4)	0.6	2.6	11.3	35.2	66.1
	KW	0.8	2.2	11.4	32.2	59.2
5						
	TRF(0)	1.8	2.0	3.9	6.6	9.8
	TRF(5)	2.4	3.9	10.7	23.0	39.4
	TRF(10)	3.0	6.1	18.5	41.4	64.6
	TRF(15)	3.6	7.9	25.9	54.6	78.2
	TRF(20)	3.9	8.9	30.2	61.8	85.6
	TRF(25)	4.0	10.5	32.5	65.1	87.8
	SWF(1.8)	3.5	9.2	30.9	63.8	87.6
	SWF(2.1)	3.7	9.2	29.7	61.5	86.4
	SWF(2.4)	3.9	9.6	28.9	59.4	84.9
	KW	4.8	10.3	28.7	57.8	81.6
10						
	TRF(0)	5.0	5.4	8.1	12.2	17.1
	TRF(5)	6.0	8.6	17.9	33.4	49.7
	TRF(10)	7.4	11.8	28.8	53.9	74.3
	TRF(15)	8.1	15.4	37.7	66.5	85.8
	TRF(20)	8.1	17.2	43.8	72.9	91.6
	TRF(25)	8.3	18.3	46.6	76.2	93.4
	SWF(1.8)	8.0	17.1	43.6	75.7	93.3
	SWF(2.1)	8.5	17.1	42.4	74.1	92.6
	SWF(2.4)	8.8	17.1	41.3	71.8	91.5
	KW	9.9	17.9	42.3	70.1	88.5

TABLE 16-(3)

SAMPLE SIZES= 10 15 15 20
DISTRIBUTION: ALL 1/U

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.3	0.4	0.9	2.4	5.3
	TRF(5)	0.3	0.6	2.2	6.5	15.6
	TRF(10)	0.6	1.2	5.5	17.2	38.2
	TRF(15)	0.7	1.6	8.9	26.4	53.1
	TRF(20)	0.9	2.4	11.0	34.2	63.2
	TRF(25)	1.1	2.5	11.9	36.1	66.1
	SWF(2.1)	0.9	2.3	10.0	31.8	63.2
	KW	1.1	2.5	9.7	29.1	54.4
5						
	TRF(0)	1.8	2.8	4.7	9.0	13.9
	TRF(5)	2.4	4.2	7.9	18.7	31.3
	TRF(10)	3.4	6.2	17.1	36.4	59.9
	TRF(15)	3.9	8.5	23.5	49.8	74.0
	TRF(20)	4.8	9.3	28.0	58.7	82.0
	TRF(25)	5.2	10.4	30.3	61.7	84.9
	SWF(2.1)	4.7	9.5	28.2	58.2	83.2
	KW	5.0	10.6	27.4	54.3	78.6
10						
	TRF(0)	5.3	7.3	9.6	15.8	21.0
	TRF(5)	6.6	9.6	15.9	28.0	41.8
	TRF(10)	7.6	12.6	26.3	49.0	69.6
	TRF(15)	8.7	15.3	35.4	61.4	81.8
	TRF(20)	9.5	17.1	40.5	69.9	88.6
	TRF(25)	9.8	17.9	43.3	73.7	91.1
	SWF(2.1)	9.9	17.6	41.1	70.4	89.8
	KW	10.7	18.5	40.3	67.6	86.7

TABLE 16-(4)

SAMPLE SIZES= 20 20 20 20 20 20
DISTRIBUTION: ALL 1/U

ALPHA	PHI 0.0	0.5	1.0	1.5	2.0
1					
TRF(0)	0.2	0.3	0.5	1.2	2.6
TRF(5)	0.3	0.9	3.1	9.8	22.8
TRF(10)	0.6	1.5	7.3	25.0	51.8
TRF(15)	0.5	2.0	11.4	39.1	70.7
TRF(20)	0.7	2.5	15.0	48.0	80.3
TRF(25)	0.8	2.8	16.6	52.0	84.8
SWF(2.1)	0.6	2.4	14.9	47.1	81.3
KW	0.8	3.0	14.8	43.5	75.0
5					
TRF(0)	1.7	2.0	2.8	5.3	8.4
TRF(5)	2.4	3.9	10.3	22.9	40.2
TRF(10)	3.0	6.4	19.9	46.0	71.4
TRF(15)	3.2	8.1	28.8	62.2	86.2
TRF(20)	3.8	9.5	34.6	71.1	92.4
TRF(25)	3.9	11.1	37.3	74.8	94.9
SWF(2.1)	3.7	9.7	34.1	71.7	93.4
KW	4.2	10.8	35.3	68.1	91.1
10					
TRF(0)	4.8	5.2	6.8	10.7	15.1
TRF(5)	5.3	8.0	17.2	33.3	50.6
TRF(10)	6.9	12.4	30.1	57.5	80.0
TRF(15)	7.8	15.7	41.4	73.1	91.4
TRF(20)	7.9	18.1	47.9	80.5	95.9
TRF(25)	8.9	19.4	51.8	84.0	97.2
SWF(2.1)	8.1	17.7	47.6	81.5	96.2
KW	9.2	19.2	48.2	79.5	95.1

TABLE 16-(5)

SAMPLE SIZES= 10 10 15 15 20 20
DISTRIBUTION: ALL 1/U

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.5	0.5	0.8	2.3	3.8
	TRF(5)	0.5	0.9	2.4	7.5	16.2
	TRF(10)	0.8	1.4	6.2	21.1	44.8
	TRF(15)	0.9	2.0	10.6	33.2	63.0
	TRF(20)	0.7	2.7	14.3	43.5	74.5
	TRF(25)	0.9	3.1	15.4	47.1	78.4
	SWF(1.8)	0.9	2.4	12.9	43.2	76.6
	SWF(2.1)	1.1	2.6	12.9	42.2	75.2
	SWF(2.4)	1.4	2.8	12.7	41.3	73.6
	KW	0.8	2.1	13.4	38.9	69.0
5						
	TRF(0)	2.4	2.7	4.4	7.8	11.1
	TRF(5)	3.0	4.5	9.4	20.4	31.3
	TRF(10)	3.7	6.0	18.7	42.0	65.0
	TRF(15)	4.6	8.0	26.9	57.4	80.7
	TRF(20)	4.6	9.8	32.2	67.5	89.4
	TRF(25)	5.2	10.6	35.7	72.0	91.8
	SWF(1.8)	4.6	9.3	33.2	68.9	91.5
	SWF(2.1)	4.9	9.7	32.4	67.4	90.4
	SWF(2.4)	5.1	9.9	32.2	65.8	89.3
	KW	4.7	9.6	33.9	66.0	87.5
10						
	TRF(0)	6.1	6.1	9.3	13.9	18.3
	TRF(5)	8.7	10.0	17.4	30.1	42.0
	TRF(10)	7.6	11.5	28.8	54.2	74.1
	TRF(15)	8.7	15.0	38.8	69.0	87.3
	TRF(20)	9.5	17.0	45.3	78.4	93.7
	TRF(25)	10.0	18.1	48.4	81.9	95.5
	SWF(1.8)	8.6	16.3	45.7	80.0	95.4
	SWF(2.1)	9.2	16.8	45.4	78.6	94.5
	SWF(2.4)	9.4	17.0	44.4	77.0	93.6
	KW	9.5	17.7	47.4	77.5	93.3

TABLE 17

Powers of some statistics for equal variances under CAUCHY

Legend

TRF(100g): Trimmed F, $F_t(g)$

SWF(k) : Sine-wave F, $F_s(k)$

KW : Kruskal-Wallis

TABLE 17-(1)

SAMPLE SIZES= 10 10 10 10

DISTRIBUTION: CAUCHY

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.2	0.4	1.0	3.2	7.1
	TRF(5)	0.2	0.4	1.4	4.5	11.0
	TRF(10)	0.3	1.0	4.8	17.0	33.5
	TRF(15)	0.3	1.0	5.8	20.6	41.1
	TRF(20)	0.3	1.6	9.4	29.9	54.6
	TRF(25)	0.4	1.8	9.7	31.7	57.7
	SWF(2.1)	0.2	1.4	8.9	28.5	54.7
	KW	0.6	2.6	9.7	26.4	44.9
5						
	TRF(0)	1.7	2.8	5.3	10.3	16.8
	TRF(5)	1.5	2.8	6.2	14.0	23.1
	TRF(10)	2.7	5.6	15.9	34.2	52.9
	TRF(15)	2.2	6.3	18.3	40.8	62.3
	TRF(20)	3.1	8.8	25.4	53.1	74.6
	TRF(25)	2.9	8.9	27.3	55.9	78.1
	SWF(2.1)	3.0	7.9	25.6	53.4	77.0
	KW	4.3	10.2	28.2	52.1	71.5
10						
	TRF(0)	4.7	6.3	10.5	16.8	23.9
	TRF(5)	3.9	6.5	11.9	21.4	32.1
	TRF(10)	5.9	11.3	25.0	45.8	63.1
	TRF(15)	5.6	11.8	29.0	53.0	71.6
	TRF(20)	7.0	15.7	37.6	64.6	82.6
	TRF(25)	7.1	16.4	41.1	68.1	85.7
	SWF(2.1)	7.0	15.4	36.8	66.0	85.0
	KW	9.2	18.6	40.6	65.4	81.9

TABLE 17-(2)

SAMPLE SIZES= 20 20 20 20
DISTRIBUTION:CAUCHY

ALPHA		PHI 0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.3	0.3	0.4	1.5	3.0
	TRF(5)	0.4	0.7	2.6	7.6	19.0
	TRF(10)	0.4	1.1	5.5	19.3	42.0
	TRF(15)	0.4	2.0	9.3	31.8	59.0
	TRF(20)	0.5	2.5	12.7	41.0	70.4
	TRF(25)	0.6	3.0	15.0	44.7	76.5
	SWF(1.8)	0.7	2.7	13.6	42.0	74.5
	SWF(2.1)	0.8	2.8	13.2	39.9	71.9
	SWF(2.4)	0.8	2.8	12.4	38.3	68.8
	KW	0.9	2.9	12.6	36.1	61.7
5						
	TRF(0)	2.0	1.9	3.0	6.2	10.1
	TRF(5)	2.3	4.0	9.0	20.4	35.9
	TRF(10)	3.1	6.0	17.6	38.8	62.2
	TRF(15)	3.5	7.8	25.0	53.8	78.0
	TRF(20)	3.7	9.3	31.5	63.9	87.0
	TRF(25)	3.9	10.4	36.2	69.7	90.5
	SWF(1.8)	4.0	10.3	33.3	67.4	89.8
	SWF(2.1)	4.0	10.1	32.1	64.8	88.3
	SWF(2.4)	4.6	9.6	30.4	62.0	86.5
	KW	4.9	10.9	31.2	60.1	82.1
10						
	TRF(0)	4.7	5.8	7.1	11.3	16.2
	TRF(5)	5.2	8.0	15.5	30.7	46.4
	TRF(10)	6.9	11.2	27.5	51.1	71.8
	TRF(15)	8.1	14.3	37.1	65.4	85.6
	TRF(20)	8.6	16.5	43.9	74.9	91.8
	TRF(25)	8.9	18.7	48.8	79.8	94.9
	SWF(1.8)	8.7	17.3	46.4	78.2	93.9
	SWF(2.1)	9.1	17.3	44.5	76.0	93.2
	SWF(2.4)	9.4	17.3	42.9	74.0	91.8
	KW	9.9	18.6	44.3	71.8	90.0

TABLE 17-(3)

SAMPLE SIZES= 10 15 15 20
DISTRIBUTION: CAUCHY

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.2	0.3	0.8	2.2	4.2
	TRF(5)	0.3	0.5	2.3	6.3	12.7
	TRF(10)	0.5	1.0	5.4	16.9	34.1
	TRF(15)	0.5	1.6	9.3	26.8	50.7
	TRF(20)	0.8	2.0	12.3	36.5	62.6
	TRF(25)	0.8	2.5	14.4	41.1	68.7
	SWF(2.1)	0.7	2.2	11.8	36.5	63.6
	KW	0.7	2.4	12.3	30.8	54.0
5						
	TRF(0)	1.9	2.7	4.7	8.2	11.0
	TRF(5)	2.2	3.6	8.7	17.3	27.4
	TRF(10)	2.8	5.1	16.7	36.0	55.1
	TRF(15)	3.4	7.2	23.7	49.9	70.4
	TRF(20)	4.1	9.1	30.6	60.6	81.0
	TRF(25)	4.2	10.5	33.7	65.8	86.0
	SWF(2.1)	4.0	9.2	30.6	61.4	83.4
	KW	4.8	10.6	30.7	57.3	77.4
10						
	TRF(0)	5.2	6.6	9.3	14.2	18.7
	TRF(5)	5.9	8.9	15.7	26.5	38.1
	TRF(10)	6.6	10.8	25.9	46.9	64.9
	TRF(15)	7.8	13.7	35.3	61.7	78.6
	TRF(20)	8.1	16.6	43.0	72.4	87.4
	TRF(25)	8.4	18.3	46.3	77.0	91.6
	SWF(2.1)	8.3	15.9	42.8	73.0	90.0
	KW	9.6	18.5	43.3	70.0	86.0

TABLE 17-(4)

SAMPLE SIZES= 20 20 20 20 20 20
DISTRIBUTION: CAUCHY

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.2	0.2	0.5	0.7	2.4
	TRF(5)	0.3	0.5	2.6	7.7	20.1
	TRF(10)	0.3	1.3	6.8	22.7	47.1
	TRF(15)	0.5	1.7	12.1	38.6	69.1
	TRF(20)	0.6	2.5	17.2	52.0	81.3
	TRF(25)	0.6	3.1	20.4	59.6	87.2
	SWF(2.1)	0.7	2.6	17.1	52.2	84.1
	KW	0.7	2.9	17.6	48.5	77.2
5						
	TRF(0)	1.6	1.8	2.8	4.5	7.5
	TRF(5)	2.1	3.6	9.2	20.0	36.6
	TRF(10)	3.1	5.8	19.3	43.3	66.8
	TRF(15)	3.8	7.9	29.7	62.1	84.1
	TRF(20)	3.9	10.4	37.2	73.3	92.4
	TRF(25)	4.2	11.6	43.0	79.6	95.3
	SWF(2.1)	4.2	10.0	36.9	75.2	94.6
	KW	4.9	12.0	38.2	72.3	91.6
10						
	TRF(0)	4.5	4.7	6.3	9.3	13.2
	TRF(5)	5.0	7.5	16.0	30.0	47.5
	TRF(10)	6.6	11.2	29.0	55.7	76.6
	TRF(15)	7.9	15.0	41.1	72.7	90.1
	TRF(20)	8.2	17.7	50.1	82.4	95.4
	TRF(25)	8.6	19.6	55.9	87.4	97.6
	SWF(2.1)	8.7	18.6	50.9	84.0	97.1
	KW	9.6	20.9	51.2	82.0	95.4

TABLE 17-(5)

SAMPLE SIZES= 10 10 15 15 20 20
DISTRIBUTION: CAUCHY

ALPHA		PHI				
		0.0	0.5	1.0	1.5	2.0
1						
	TRF(0)	0.3	0.3	0.8	1.7	3.7
	TRF(5)	0.5	0.7	2.0	5.9	14.4
	TRF(10)	0.6	1.4	5.6	20.7	42.8
	TRF(15)	0.7	1.7	10.0	34.4	62.8
	TRF(20)	1.1	2.5	14.9	46.6	76.2
	TRF(25)	0.9	2.5	17.9	53.0	82.1
	SWF(1.8)	0.9	2.8	15.6	49.2	80.9
	SWF(2.1)	1.1	2.9	14.9	47.2	78.6
	SWF(2.4)	1.3	2.9	14.5	44.9	76.8
	KW	0.8	3.1	14.6	43.3	71.2
5						
	TRF(0)	2.2	2.3	4.0	6.6	9.8
	TRF(5)	2.8	3.8	8.6	18.0	29.5
	TRF(10)	3.4	5.8	17.3	40.3	62.7
	TRF(15)	4.0	7.9	26.1	56.4	80.0
	TRF(20)	4.1	9.8	34.2	68.7	89.0
	TRF(25)	4.5	10.9	39.1	75.1	92.6
	SWF(1.8)	4.1	10.2	35.8	72.2	92.2
	SWF(2.1)	4.6	10.3	34.7	70.4	90.8
	SWF(2.4)	4.6	10.3	33.0	68.8	89.5
	KW	4.4	11.1	35.9	68.3	88.1
10						
	TRF(0)	5.3	5.7	8.3	12.8	16.7
	TRF(5)	7.1	8.9	16.1	27.3	38.9
	TRF(10)	7.0	11.0	26.6	52.1	72.6
	TRF(15)	7.9	14.4	38.6	67.3	86.1
	TRF(20)	8.6	17.4	47.0	78.4	93.1
	TRF(25)	8.9	19.1	52.7	83.7	95.6
	SWF(1.8)	7.8	17.0	47.9	82.0	95.7
	SWF(2.1)	8.2	17.9	47.0	80.4	94.9
	SWF(2.4)	8.8	18.0	45.7	78.4	93.9
	KW	9.6	19.5	49.9	79.1	93.6

4.2 THE PERFORMANCE OF THE TRIMMED W AND F* AND THE SINE-WAVE W AND F*

The performance of the proposed statistics for unequal variances was evaluated by Monte Carlo simulations. For comparison purposes, the asymptotic distribution-free Studentized Brown and Mood test (denoted by SBM) proposed by Sen [24] was also included in the study. This statistic is defined by

$$SBM = 4\{\sum_{i=1}^{c} (m_i - n_i/2)^2/n_i - [\sum_{i=1}^{c} \hat{\delta}_i^{-1}(m_i - n_i/2)]^2/\hat{A}\}$$

where m_i, $i = 1, 2, \ldots, c$, is the number of observations in the i-th sample with values not greater than the median of the c samples pooled together, and $\hat{A} = \sum_i (n_i/\hat{\delta}_i^2)$, $\hat{\delta}_i$ is any consistent estimate of some scale parameter δ_i. The statistic SBM is asymptotically chi-square distributed with c - 1 degrees of freedom provided that

1. the density function $f(x) = F'(x)$ is continuous in a neighborhood of $x = 0$, and $f(0) > 0$ (the distribution function of i-th gruop is $F_i(x) = F[(x - \mu_i)/\delta_i]$); and
2. $N \longrightarrow \infty$ with $n_i/N = c_i$, $0 < c_i < 1$ and $\sum_i c_i = 1$.

Obviously, the performance of SBM depends on the definition of δ_i and the choice of its estimate $\hat{\delta}_i$. Following Sen's suggestion [24], we used

$$\hat{\delta}_i = [\sum_{j=1}^{[n_i/2]} (n_i/2 - j)^3 Z_{ij}]/(n_i/2)^4$$

as an estimate of

$$\delta_i = \Big[\sum_{j=1}^{[n_i/2]} (n_i/2 - j)^3 E(Z_{ij})\Big]/(n_i/2)^4$$

where $Z_{ij} = X_{i(n_i-j+1)} - X_{i(j)}$ for $j = 1, 2, \ldots, [n_i/2]$, $i = 1, 2, \ldots, c$ and $X_{i(j)}$ is the j-th ordered value in the i-th sample.

4.2.1 Distributions Used in the Study

The following distributions were used in the study:

Tag	Distribution
NORMAL	Standard normal, N(0, 1)
10% 5N	Mixture of two normals, 0.9N(0, 1) + 0.1N(0, 25)
10% 10N	Mixture of two normals, 0.9N(0, 1) + 0.1N(0, 100)
25% 1/U	Mixture of normal and normal/uniform 0.25N(0, 1) + 0.75N(0, 1)/U[0, 1]
CAUCHY	Cauchy, C(0, 1)

These distributions will be referred to, as before, by their tag names.

4.2.2 Sampling Situations

The word "sampling situation" has the same meaning as in Sec 4.1.2 except for one added factor $(\sigma_i) = (r_i\sigma)$ where σ is the standard deviation of the underlying distribution or $\sigma = F^{-1}(0.8413)$ for a distribution with infinite variance (see Section 4.1.2). This factor is needed in the unequal variances situation. The number of groups studied was $c = 4$ and the sample sizes were $(n_i) = (10, 15, 15, 20)$. Table 18 shows the various sampling situations considered in the evaluation.

TABLE 18

Sampling situations

Distributions	$(r_i) = (\sigma_i/\sigma)$	(μ_i/σ)
NORMAL	1,1,1,1	(0,0,0,0), (1,0,0,0), (0,0,0,.7), (.5,0,0,.5)
	1,2,2,3	(0,0,0,0), (1,0,0,0), (0,0,0,1), (1,0,0,1)
	3,2,2,1	(0,0,0,0), (1,0,0,0), (0,0,0,1), (1,0,0,1)
10% 5N	1,1,1,1	(0,0,0,0), (1,0,0,0), (0,0,0,.7), (.5,0,0,.5)
	1,2,2,3	(0,0,0,0), (1,0,0,0), (0,0,0,1), (1,0,0,1)
	3,2,2,1	(0,0,0,0), (1,0,0,0), (0,0,0,1), (1,0,0,1)
10% 10N	1,1,1,1	(0,0,0,0), (.3,0,0,0), (0,0,0,.3), (.3,0,0,.3)
	1,2,2,3	(0,0,0,0), (.5,0,0,0), (0,0,0,.5), (.5,0,0,.5)
	3,2,2,1	(0,0,0,0), (.5,0,0,0), (0,0,0,.5), (.5,0,0,.5)
25% 1/U	1,1,1,1	(0,0,0,0), (1,0,0,0), (0,0,0,.7), (.5,0,0,.5)
	1,2,2,3	(0,0,0,0), (1,0,0,0), (0,0,0,1), (1,0,0,1)
	3,2,2,1	(0,0,0,0), (1,0,0,0), (0,0,0,1), (1,0,0,1)
CAUCHY	1,1,1,1	(0,0,0,0), (1,0,0,0), (0,0,0,.7), (.5,0,0,.5)
	1,2,2,3	(0,0,0,0), (1,0,0,0), (0,0,0,1), (1,0,0,1)
	3,2,2,1	(0,0,0,0), (1,0,0,0), (0,0,0,1), (1,0,0,1)

4.2.3 Sampling Method

For each sampling situation, a set of n_i random observations was generated and allocated to the i-th group in the same way as described in Section 4.1.3 from the designated distribution. Each observation allocated to the i-th group was multiplied by σ_i and then increased by μ_i. All statistics were then computed and checks were made to count the number of rejections of the null hypothesis of equal group means. This procedure was again repeated 5,000 times and the empirical power was obtained in the same way as before for α = 1, 5, 10%.

As mentioned in the previous chapters, $W_t(0.2)$, $W_t(0.25)$ and $W_S(1.8)$ were excluded in the comparison here because of their somewhat excessive liberal tendency for some situations under normality.

4.2.4 Results

Tables 24 - 28 show the results of the simulations. We present summary tables (Tables 19 - 23) of selected statistics for each distribution considered with the ranking of the best 4 statistics based on the case of $\alpha = 5\%$. The ranking could be different for other values of alpha. We do not emphasize small differences in power but try to understand the general trend. For further details, the reader is referred to Tables 24 - 28.

From now on we assume that $\sigma = 1$ without loss of generality.

In Tables 19 - 23, the best g for $W_t(g)$ and $F_t^*(g)$ and the best k for $W_S(k)$ and $F_S^*(k)$ under a given distribution are sometimes different from one situation to another. For instance, under 25% 1/U, the best g for $F_t^*(g)$ with $\alpha = 5\%$ is 0.2 for $(\mu_i) = (1, 0, 0, 0)$, but is 0.15 for $(\mu_i) = (1, 0, 0, 1)$, etc. But their performances are very close to each other in power. In this sense, it does not cause a major problem to select a good value of g or k for a given distribution. The best g or k for the null cases in Tables 19 - 23 are, to our judgement, the best ones for the distribution.

Note that the power here is not depicted as a function of some sort of noncentrality parameter as in the equal variances case but it is obtained for individual combinations of σ_i's, μ_i's and n_i's. We like to see how these statistics compared under various combinations of parameters rather than to obtain a power curve.

TABLE 19

Powers of selected statistics for unequal variances

Under NORMALITY (α = 5%)
(n_i) = (10, 15, 15, 20)

(σ_i)	(μ_i)	W	F*	SBM	Best $W_t(g)$ g	power	Best $F^*_t(g)$ g	power	Best $W_s(k)$ k	power	Best $F^*_s(k)$ k	power
1,1,1,1	0,0,0,0	4.7	4.8	4.7	0	4.7	0	4.8	2.4	4.9	2.4	4.6
	1,0,0,0	60.5	63.5	38.4	0	60.5	0	63.5	2.4	55.8	2.4	58.3
	0,0,0,.7	48.2	51.3	34.2	0	48.2	0	51.3	2.4	41.9	2.4	46.2
	.5,0,0,.5	30.1	32.0	19.4	0	30.1	0	32.0	2.4	25.7	2.4	27.5
1,2,2,3	0,0,0,0	5.0	5.6	3.9	0	5.0	0	5.6	2.4	4.9	2.4	5.4
	1,0,0,0	39.0	16.0	18.3	0	39.0	0	16.0	2.4	34.6	2.4	13.9
	0,0,0,1	18.4	28.0	11.4	0	18.4	0	28.0	2.4	15.7	2.4	24.5
	1,0,0,1	37.1	28.3	20.6	0	37.1	0	28.3	2.4	32.2	2.4	24.7
3,2,2,1	0,0,0,0	5.0	5.6	5.1	0	5.0	0	5.6	2.4	5.3	2.4	5.7
	1,0,0,0	11.6	16.6	8.7	0	11.6	0	16.6	2.4	11.2	2.4	15.2
	0,0,0,1	44.6	21.2	34.2	0	44.6	0	21.2	2.4	38.5	2.4	19.2
	1,0,0,1	41.5	24.3	30.2	0	41.5	0	24.3	2.4	34.6	2.4	21.4

The best 4 statistics

(σ_i)	(μ_i)	
1,1,1,1	1,0,0,0	$F^* > F^*_t(0.05) > W > F^*_t(0.1)$
	0,0,0,.7	$F^* > F^*_t(0.05) > W > F^*_t(0.1)$
	.5,0,0,.5	$F^* > W > F^*_t(0.05) > W_t(0.05)$
1,2,2,3	1,0,0,0	$W > W_t(0.05) > W_t(0.1) > W_t(0.15)$
	0,0,0,1	$F^* > F^*_t(0.05) > F^*_t(0.1) > F^*_t(0.15)$
	1,0,0,1	$W > W_t(0.05) > W_t(0.1) > W_t(0.15)$
3,2,2,1	1,0,0,0	$F^* > F^*_t(0.05) > F^*_t(0.1) > F^*_s(2.4)$
	0,0,0,1	$W > W_t(0.05) > W_t(0.1) > W_t(0.15)$
	1,0,0,1	$W > W_t(0.05) > W_t(0.1) > W_t(0.15)$

TABLE 20

Powers of selected statistics for unequal variances

Under 10% 5N (α = 5%)
(n_i) = (10, 15, 15, 20)

(σ_i)	(μ_i)	W	F*	SBM	Best $W_t(g)$ g	power	Best $F_t^*(g)$ g	power	Best $W_s(k)$ k	power	Best $F_s^*(k)$ k	power
1,1,1,1	0,0,0,0	3.2	3.3	3.9	.15	4.5	.15	4.5	2.4	4.6	2.4	3.9
	1,0,0,0	74.9	67.3	72.2	.15	89.3	.15	92.7	2.4	90.3	2.4	92.6
	0,0,0,.7	63.8	55.9	75.9	.15	86.1	.15	87.2	2.4	86.1	2.1	87.6
	.5,0,0,.5	45.9	35.9	51.1	.15	62.9	.15	63.1	2.4	63.3	2.4	62.9
1,2,2,3	0,0,0,0	2.8	4.1	4.2	.15	4.4	.15	5.6	2.4	4.6	2.4	4.9
	1,0,0,0	56.3	19.4	40.9	.15	75.4	.15	35.4	2.4	75.7	2.4	34.2
	0,0,0,1	24.2	31.0	25.3	.1	34.7	.15	49.8	2.4	34.4	2.4	49.7
	1,0,0,1	55.0	35.0	55.7	.15	73.1	.15	58.2	2.4	73.6	2.4	57.2
3,2,2,1	0,0,0,0	2.9	3.3	4.8	.15	4.9	.15	5.0	2.4	4.8	2.4	4.8
	1,0,0,0	18.1	20.9	18.2	.15	22.4	.15	30.3	2.4	22.3	2.4	30.2
	0,0,0,1	63.2	30.3	77.3	.15	83.2	.15	50.4	2.4	83.2	2.4	50.5
	1,0,0,1	59.7	33.5	72.3	.15	79.3	.15	52.5	2.4	79.3	2.4	52.9

The best 4 statistics

1,1,1,1	1,0,0,0	$F_t^*(.15) > F_t^*(.2) > F_s^*(2.4) > F_s^*(2.1)$
	0,0,0,.7	$F_s^*(2.1) > F_s^*(2.4) > F_t^*(.15) > F_s^*(1.8)$
	.5,0,0,.5	$W_s(2.4) > F_t^*(.15) > F_s^*(2.4) > W_t(.15)$
1,2,2,3	1,0,0,0	$W_s(2.4) > W_t(.15) > W_s(2.1) > W_t(.1)$
	0,0,0,1	$F_t^*(.15) > F_s^*(2.4) > F_s^*(2.1) > F_t^*(.2)$
	1,0,0,1	$W_s(2.4) > W_t(.15) > W_t(.1) > W_s(2.1)$
3,2,2,1	1,0,0,0	$F_t^*(.15) > F_s^*(2.4) > F_t^*(.1) > F_t^*(.2)$
	0,0,0,1	$W_s(2.4) > W_t(.15) > W_t(.1) > W_s(2.1)$
	1,0,0,1	$W_s(2.4) > W_t(.15) > W_t(.1) > W_s(2.1)$

TABLE 21

Powers of selected statistics for unequal variances

Under 10% 10N ($\alpha = 5\%$)
$(n_i) = (10, 15, 15, 20)$

(σ_i)	(μ_i)	W	F*	SBM	Best $W_t(g)$ g	power	Best $F_t^*(g)$ g	power	Best $W_S(k)$ k	power	Best $F_S^*(k)$ k	power
1,1,1,1	0,0,0,0	1.6	2.3	3.1	.15	4.0	.2	4.2	2.4	4.7	2.4	4.1
	.3,0,0,0	22.3	9.2	24.7	.15	39.9	.2	40.1	2.4	45.6	2.4	44.4
	0,0,0,.3	23.7	14.8	48.1	.15	59.3	.2	59.9	2.4	64.4	2.4	65.8
	.3,0,0,.3	31.2	18.1	53.4	.15	67.2	.2	65.0	2.4	71.9	2.4	71.6
1,2,2,3	0,0,0,0	1.6	2.4	3.3	.15	4.3	.2	5.2	2.4	4.2	2.4	4.0
	.5,0,0,0	29.2	7.0	31.9	.15	60.8	.2	26.3	2.4	67.5	2.4	28.0
	0,0,0,.5	10.8	12.8	20.8	.15	28.4	.2	41.2	2.4	30.6	2.4	44.0
	.5,0,0,.5	28.4	13.0	42.8	.15	58.3	.2	44.7	2.4	64.7	2.4	48.8
3,2,2,1	0,0,0,0	2.1	2.3	4.4	.15	4.7	.2	5.3	2.4	4.9	2.4	5.2
	.5,0,0,0	10.4	7.4	13.5	.15	16.7	.2	23.3	2.4	18.5	2.4	24.9
	0,0,0,.5	29.3	11.1	64.3	.15	70.0	.2	36.1	2.4	75.4	2.4	40.2
	.5,0,0,.5	28.2	12.1	58.1	.15	65.6	.2	39.3	2.4	70.1	2.4	43.5

The best 4 statistics

1,1,1,1	.3,0,0,0	$W_S(2.4) > F_S^*(2.4) > W_S(2.1) > F_S^*(2.1)$
	0,0,0,.3	$F_S^*(2.4) > F_S^*(2.1) > W_S(2.4) > F_S^*(1.8)$
	.3,0,0,.3	$W_S(2.4) > F_S^*(2.4) > F_S^*(2.1) > F_S^*(1.8)$
1,2,2,3	.5,0,0,0	$W_S(2.4) > W_S(2.1) > W_t(.15) > W_t(.1)$
	0,0,0,.5	$F_S^*(2.4) > F_S^*(2.1) > F_S^*(1.8) > F_t^*(.2)$
	.5,0,0,.5	$W_S(2.4) > W_S(2.1) > W_t(.15) > W_t(.1)$
3,2,2,1	.5,0,0,0	$F_S^*(2.4) > F_S^*(2.1) > F_t^*(.2) > F_S^*(1.8)$
	0,0,0,.5	$W_S(2.4) > W_S(2.1) > W_t(.15) > W_t(.1)$
	.5,0,0,.5	$W_S(2.4) > W_S(2.1) > W_t(.15) > W_t(.1)$

TABLE 22

Powers of selected statistics for unequal variances

Under 25% 1/U (α = 5%)
(n_i) = (10, 15, 15, 20)

(σ_i)	(μ_i)	W	F*	SBM	Best $W_t(g)$ g	power	Best $F^*_t(g)$ g	power	Best $W_S(k)$ k	power	Best $F^*_S(k)$ k	power
1,1,1,1	0,0,0,0	2.6	1.9	4.0	.15	4.4	.2	4.7	2.4	4.2	2.4	4.3
	1,0,0,0	36.5	16.9	33.0	.1	52.3	.15	53.5	2.4	55.2	2.4	54.8
	0,0,0,.7	22.2	13.4	32.7	.15	42.0	.15	42.8	2.4	42.1	2.4	43.6
	.5,0,0,.5	14.0	8.0	19.1	.15	25.4	.2	26.1	2.4	25.4	2.4	26.1
1,2,2,3	0,0,0,0	2.7	2.4	3.5	.15	4.8	.2	5.1	2.4	4.6	2.4	4.9
	1,0,0,0	20.2	5.0	16.7	.15	33.2	.2	13.2	2.4	34.2	2.4	12.6
	0,0,0,1	7.7	8.4	10.7	.15	14.2	.15	22.6	2.4	13.6	2.4	22.0
	1,0,0,1	18.1	7.9	19.8	.15	31.0	.15	23.7	2.4	31.7	2.4	22.9
3,2,2,1	0,0,0,0	2.6	1.9	4.7	.15	5.0	.2	5.5	2.4	4.9	2.4	5.1
	1,0,0,0	6.3	4.7	8.3	.15	9.7	.2	13.7	2.4	10.2	2.4	13.1
	0,0,0,1	19.5	6.4	33.4	.15	38.3	.2	18.7	2.4	38.6	2.4	17.2
	1,0,0,1	19.1	8.0	30.5	.15	35.0	.25	20.6	2.4	35.9	2.4	20.6

The best 4 statistics

1,1,1,1	1,0,0,0	$W_S(2.4) > F^*_S(2.4) > F^*_S(2.1) > F^*_t(.15)$
	0,0,0,.7	$F^*_S(2.4) > F^*_S(2.1) > F^*_t(.15) > F^*_t(.2)$
	.5,0,0,.5	$F^*_t(.2) > F^*_S(2.4) > F^*_t(.15) > W_S(2.4)$
1,2,2,3	1,0,0,0	$W_S(2.4) > W_t(.15) > W_S(2.1) > W_t(.1)$
	0,0,0,1	$F^*_t(.15) > F^*_t(.2) > F^*_S(2.4) > F^*_t(.25)$
	1,0,0,1	$W_S(2.4) > W_t(.15) > W_S(2.1) > W_t(.1)$
3,2,2,1	1,0,0,0	$F^*_t(.2) > F^*_t(.15) > F^*_t(.25) > F^*_S(2.4)$
	0,0,0,1	$W_S(2.4) > W_t(.15) > W_t(.1) > W_S(2.1)$
	1,0,0,1	$W_S(2.4) > W_t(.15) > W_S(2.1) > W_t(.1)$

TABLE 23

Powers of selected statistics for unequal variances

Under CAUCHY (α = 5%)
(n_i) = (10, 15, 15, 20)

(σ_i)	(μ_i)	W	F*	SBM	Best $W_t(g)$ g	Best $W_t(g)$ power	Best $F_t^*(g)$ g	Best $F_t^*(g)$ power	Best $W_S(k)$ k	Best $W_S(k)$ power	Best $F_S^*(k)$ k	Best $F_S^*(k)$ power
1,1,1,1	0,0,0,0	1.5	1.3	3.7	.15	2.1	.25	3.1	2.1	2.9	1.8	3.2
	1,0,0,0	18.2	5.4	34.5	.15	50.2	.25	57.2	2.1	60.1	1.8	54.3
	0,0,0,.7	9.0	4.8	47.7	.15	41.1	.25	45.8	2.1	47.1	1.8	43.7
	.5,0,0,.5	6.1	3.5	30.3	.15	23.0	.25	26.0	2.1	29.7	1.8	25.1
1,2,2,3	0,0,0,0	0.9	1.6	3.4	.15	2.0	.25	3.5	2.1	2.9	1.8	3.6
	1,0,0,0	9.2	2.8	19.6	.15	30.7	.25	15.0	2.1	38.5	1.8	12.7
	0,0,0,1	3.6	3.7	17.1	.15	12.9	.25	24.3	2.1	15.8	1.8	22.4
	1,0,0,1	8.4	3.2	31.6	.15	29.9	.25	27.9	2.1	37.6	2.4	24.9
3,2,2,1	0,0,0,0	1.3	1.3	4.5	.15	2.1	.25	3.3	2.1	3.4	1.8	3.1
	1,0,0,0	3.8	2.1	10.8	.15	8.6	.25	13.1	2.4	12.3	2.4	13.2
	0,0,0,1	8.1	2.8	46.9	.15	35.1	.25	17.7	2.1	43.8	2.4	17.7
	1,0,0,1	8.0	3.2	44.2	.15	33.0	.25	21.3	2.1	41.1	1.8	19.9

The best 4 statistics

1,1,1,1	1,0,0,0	$W_S(2.1) > W_S(2.4) > F_t^*(.25) > F_S^*(1.8)$
	0,0,0,1	$SBM > W_S(2.1) > W_S(2.4) > F_t^*(.25)$
	.5,0,0,.5	$SBM > W_S(2.1) > W_S(2.4) > F_t^*(.25)$
1,2,2,3	1,0,0,0	$W_S(2.1) > W_S(2.4) > W_t(.15) > W_t(.1)$
	0,0,0,1	$F_t^*(.25) > F_S^*(1.8) > F_t^*(.2) > F_S^*(2.1)$
	1,0,0,1	$W_S(2.1) > W_S(2.4) > SBM > W_t(.15)$
3,2,2,1	1,0,0,0	$F_S^*(2.4) > F_t^*(.25) > F_S^*(1.8) > F_S^*(2.1)$
	0,0,0,1	$SBM > W_S(2.1) > W_S(2.4) > W_t(.15)$
	1,0,0,1	$SBM > W_S(2.1) > W_S(2.4) > W_t(.15)$

4.2.5 General Conclusions and Recommendations

Summarizing the results of the experiment, we draw some general conclusions as follows:

1. W and F* performed poorly except under normality. Under long-tailed distributions, they are conservative and have very low power.
2. The trimmed statistics performed fairly well for long-tailed distributions. The best trimming proportion (g) changes from one situation to another. The key factor in determining the best g is the tail-length of the underlying distribution. The general rule is that the longer the tail, the larger the trimming proportion should be. For extremely long-tailed distributions such as CAUCHY, the restriction on g $(0 \leq g \leq 0.15)$ for $W_t(g)$ weakened its performance. For other distributions considered, however, this restriction caused no or only slight loss of power. Trimmed statistics with small values of g are also conservative under long-tailed distributions.
3. The sine-wave statistics performed quite well throughout all distributions except normality. As before, 2.4 is the best value for k in $W_S(k)$ and $F^*_S(k)$ for normal and moderately long-tailed distributions such as 10% 5N, 25% 1/U and 10% 10N. For the extremely long-tailed distribution such as CAUCHY, k = 1.8 is the best choice for $F^*_S(k)$ and k = 2.1 for $W_S(k)$ (it would also be 1.8 if there were not restriction on k for $W_S(k)$). $F^*_S(2.1)$ is always between $F^*_S(2.4)$ and $F^*_S(1.8)$. In any case, power differences among $F^*_S(k)$'s and among $W_S(k)$'s are not great.

4. The Studentized Brown and Mood test (SBM) was never among the best four for all distributions except CAUCHY under which SBM performed quite well. This is not surprising because the unmodified Brown and Mood test is known to be good for extremely long-tailed distributions.

5. It is stated in Brown and Forsythe [4] that under normality W is more powerful than F* when extreme means have small variances and conversely for extreme means with large variances (these alternatives are denoted by H(2) or H(3) in Tables 24 - 28). The same was expected for the W-type statistics (trimmed and sine-wave W) and the F*-type statistics (trimmed and sine-wave F*). For the balanced alternative such that one extreme mean is with a small variance and the other is with a large variance (H(4) in Tables 24 - 28), the W-type is more powerful than the F*-type. When variances are equal, the F*-type statistics are slightly superior to the W-type ones for NORMAL and 10% 5N and conversely for CAUCHY but they are very close in power for 10% 10N and 25% 1/U. Note that here we are just comparing $W_t(g)$ and $F^*_t(g)$ with the same g and $W_s(k)$ and $F^*_s(k)$ with the same k.

Consequently, we give the following recommendations:

1. When extreme means have small variances or the alternative is balanced, according to Point 5 above, the W-type statistic should be used except for CAUCHY where SBM can be used. The choice among the W-type statistics depends on the underlying distribution. For normal or close to normal distributions, W or $W_t(.05)$ is recommended; for moderately long-tailed distributions

such as 10% 5N, 10% 10N and 25% 1/U, $W_S(2.4)$ or $W_t(0.15)$; and for extremely long-tailed distributions such as CAUCHY, SBM or $W_S(2.1)$. Here, it is assumed that some information about the tail-length of the parent distribution is known. When, however, we have no idea about the tail-length, $W_S(2.1)$ or $W_S(2.4)$ is recommended because of their excellent overall performance.

2. When extreme means have large variances, the F*-type statistics are recommended for all distributions again by Point 5 above. Assuming the tail-length of the underlying distribution is known, F^* or $F_t^*(0.05)$ is recommended for normal or close to normal distributions; $F_S^*(2.4)$ or $F_t^*(.15)$ or $F_t^*(.2)$ for moderately long-tailed cases; and $F_t^*(.25)$ or $F_S^*(1.8)$ for extremely long-tailed ones. If no information about the tail-length is available, $F_S^*(2.1)$ is recommended.

TABLE 24

Powers of some statistics for unequal variances under normality

Legend

TRW(100g) : Trimmed-W, $W_t(g)$
TRF*(100g): Trimmed F*, $F_t^*(g)$
SWW(k) : Sine-wave W, $W_s(k)$
SWF*(k) : Sine-wave F*, $F_s^*(k)$
SBM : Studentized Brown and Mood

TABLE 24-(1)

DISTRIBUTION:NORMAL
SAMPLE SIZES= 10 15 15 20
SIGMAS= 1.0 1.0 1.0 1.0
MEANS
H(1): 0.0 0.0 0.0 0.0
H(2): 1.0 0.0 0.0 0.0
H(3): 0.0 0.0 0.0 0.7
H(4): 0.5 0.0 0.0 0.5

ALPHA	H(1)	H(2)	H(3)	H(4)
1				
TRW (0)	0.9	34.9	23.6	11.4
TRW (5)	1.0	33.7	21.7	10.7
TRW (10)	1.1	31.7	20.1	10.4
TRW (15)	1.2	30.0	18.1	9.2
TRF*(0)	1.0	36.8	25.9	12.6
TRF*(5)	1.0	35.7	25.4	12.4
TRF*(10)	1.1	33.4	23.0	11.2
TRF*(15)	1.0	31.0	21.1	10.1
TRF*(20)	0.8	27.1	18.9	9.2
TRF*(25)	0.8	24.1	17.0	8.4
SWW (2.1)	1.2	25.8	14.3	7.4
SWW (2.4)	1.1	29.5	16.4	8.7
SWF*(1.8)	0.9	25.0	17.4	8.2
SWF*(2.1)	0.8	27.9	19.3	8.8
SWF*(2.4)	0.9	30.8	21.2	10.0
SBM	0.9	13.6	14.7	6.6

TABLE 24-(1) (continued)

ALPHA		H(1)	H(2)	H(3)	H(4)
5					
	TRW (0)	4.7	60.5	48.2	30.1
	TRW (5)	4.9	59.6	46.7	29.0
	TRW (10)	5.2	57.0	45.0	27.4
	TRW (15)	5.5	54.8	42.6	26.2
	TRF*(0)	4.8	63.5	51.3	32.0
	TRF*(5)	4.9	62.6	50.0	30.0
	TRF*(10)	4.9	59.9	47.2	28.3
	TRF*(15)	5.0	57.2	45.1	27.0
	TRF*(20)	5.0	53.3	41.9	25.1
	TRF*(25)	5.0	50.5	39.9	24.1
	SWW (2.1)	5.0	53.3	38.9	23.9
	SWW (2.4)	4.9	55.8	41.9	25.7
	SWF*(1.8)	4.5	53.4	40.6	24.1
	SWF*(2.1)	4.3	55.9	43.9	26.0
	SWF*(2.4)	4.6	58.3	46.2	27.5
	SBM	4.7	38.4	34.2	19.4
10					
	TRW (0)	9.4	72.9	62.5	43.7
	TRW (5)	10.0	72.7	61.3	42.4
	TRW (10)	10.4	69.9	59.5	40.6
	TRW (15)	10.5	68.5	57.7	38.9
	TRF*(0)	9.2	75.8	65.6	44.5
	TRF*(5)	9.6	75.0	63.8	44.0
	TRF*(10)	9.7	72.7	61.5	41.8
	TRF*(15)	9.7	71.4	59.8	39.7
	TRF*(20)	9.7	67.8	56.2	37.9
	TRF*(25)	10.0	64.9	54.6	36.3
	SWW (2.1)	10.0	67.5	55.2	37.7
	SWW (2.4)	9.9	69.5	57.6	39.1
	SWF*(1.8)	9.6	67.4	56.5	37.6
	SWF*(2.1)	9.5	70.4	59.1	39.5
	SWF*(2.4)	9.4	72.2	60.8	41.1
	SBM	8.9	52.0	48.1	31.6

TABLE 24-(2)

DISTRIBUTION:NORMAL
SAMPLE SIZES= 10 15 15 20
SIGMAS= 1.0 2.0 2.0 3.0
MEANS
H(1): 0.0 0.0 0.0 0.0
H(2): 1.0 0.0 0.0 0.0
H(3): 0.0 0.0 0.0 1.0
H(4): 1.0 0.0 0.0 1.0

ALPHA		H(1)	H(2)	H(3)	H(4)
1					
	TRW (0)	0.9	17.1	5.6	15.8
	TRW (5)	1.0	17.0	5.2	14.8
	TRW (10)	0.9	15.5	4.8	13.8
	TRW (15)	1.1	14.3	4.6	13.1
	TRF*(0)	1.6	4.5	12.9	12.5
	TRF*(5)	1.7	4.4	12.1	11.8
	TRF*(10)	1.7	4.5	11.6	11.2
	TRF*(15)	1.7	4.5	11.0	10.5
	TRF*(20)	1.7	4.0	10.0	9.3
	TRF*(25)	1.7	3.7	9.3	8.9
	SWW (2.1)	1.2	11.5	3.6	10.2
	SWW (2.4)	1.1	13.5	3.9	11.7
	SWF*(1.8)	1.3	2.8	8.3	8.1
	SWF*(2.1)	1.3	3.2	9.1	8.9
	SWF*(2.4)	1.4	3.4	10.0	9.6
	SBM	0.8	4.2	2.6	6.7
5					
	TRW (0)	5.0	39.0	18.4	37.1
	TRW (5)	5.1	38.0	17.6	36.3
	TRW (10)	5.0	35.7	17.2	34.3
	TRW (15)	5.1	35.0	16.5	33.1
	TRF*(0)	5.6	16.0	28.0	28.3
	TRF*(5)	5.8	15.4	27.3	27.4
	TRF*(10)	5.7	14.7	26.5	26.7
	TRF*(15)	6.1	14.4	24.8	25.4
	TRF*(20)	6.2	13.7	23.0	23.5
	TRF*(25)	6.5	13.0	22.4	22.4
	SWW (2.1)	5.0	32.1	14.8	30.6
	SWW (2.4)	4.9	34.6	15.7	32.2
	SWF*(1.8)	5.2	12.4	22.0	21.8
	SWF*(2.1)	5.1	12.9	23.2	23.2
	SWF*(2.4)	5.4	13.9	24.5	24.7
	SBM	3.9	18.3	11.4	20.6

TABLE 24-(2) (continued)

ALPHA 10	H(1)	H(2)	H(3)	H(4)
TRW (0)	9.9	52.7	28.3	50.8
TRW (5)	10.0	51.1	27.6	50.2
TRW (10)	9.9	49.2	27.7	48.6
TRW (15)	10.1	48.1	26.7	47.0
TRF*(0)	10.8	27.2	37.6	39.7
TRF*(5)	11.0	26.7	37.0	39.1
TRF*(10)	10.9	26.0	36.1	38.1
TRF*(15)	10.7	24.7	34.5	37.3
TRF*(20)	10.7	23.7	33.0	35.1
TRF*(25)	11.2	23.5	32.0	33.8
SWW (2.1)	9.8	46.8	24.9	44.5
SWW (2.4)	9.8	48.3	25.8	46.4
SWF*(1.8)	10.0	22.4	32.3	34.0
SWF*(2.1)	10.0	23.3	33.4	35.5
SWF*(2.4)	10.2	24.2	34.6	36.7
SBM	8.4	31.3	20.3	34.6

TABLE 24-(3)

DISTRIBUTION:NORMAL
SAMPLE SIZES= 10 15 15 20
SIGMAS= 3.0 2.0 2.0 1.0
MEANS
H(1): 0.0 0.0 0.0 0.0
H(2): 1.0 0.0 0.0 0.0
H(3): 0.0 0.0 0.0 1.0
H(4): 1.0 0.0 0.0 1.0

ALPHA	H(1)	H(2)	H(3)	H(4)
1				
TRW (0)	1.1	3.1	20.7	17.3
TRW (5)	1.1	3.2	18.9	16.3
TRW (10)	1.2	3.5	17.4	14.8
TRW (15)	1.3	3.6	15.6	13.6
TRF*(0)	1.4	7.0	7.3	8.8
TRF*(5)	1.4	6.4	6.9	8.3
TRF*(10)	1.5	6.1	6.9	8.2
TRF*(15)	1.6	5.8	6.5	7.9
TRF*(20)	1.7	5.9	6.4	7.9
TRF*(25)	1.7	5.3	5.9	7.1
SWW (2.1)	1.6	3.1	12.1	10.3
SWW (2.4)	1.4	3.2	14.3	12.1
SWF*(1.8)	1.4	4.9	5.0	5.9
SWF*(2.1)	1.4	5.1	5.3	6.3
SWF*(2.4)	1.4	5.7	6.1	7.2
SBM	0.8	1.9	14.3	12.0
5				
TRW (0)	5.0	11.6	44.6	41.5
TRW (5)	5.2	11.5	42.9	39.6
TRW (10)	5.5	11.9	41.2	37.4
TRW (15)	5.5	11.7	39.1	35.6
TRF*(0)	5.6	16.6	21.2	24.3
TRF*(5)	5.8	16.1	21.4	23.9
TRF*(10)	5.9	15.8	20.4	23.0
TRF*(15)	6.0	15.1	19.8	21.9
TRF*(20)	6.2	15.1	18.7	20.9
TRF*(25)	6.0	14.7	18.0	20.0
SWW (2.1)	5.7	10.8	35.7	32.3
SWW (2.4)	5.3	11.2	38.5	34.6
SWF*(1.8)	5.8	13.5	17.1	19.3
SWF*(2.1)	5.8	14.2	18.2	20.5
SWF*(2.4)	5.7	15.2	19.2	21.4
SBM	5.1	8.7	34.2	30.2

TABLE 24-(3) (continued)

ALPHA 10	H(1)	H(2)	H(3)	H(4)
TRW (0)	10.0	19.0	59.4	56.0
TRW (5)	10.1	19.1	57.3	55.3
TRW (10)	10.3	19.0	55.8	52.8
TRW (15)	10.5	19.7	53.9	50.3
TRF*(0)	10.4	24.7	34.2	37.2
TRF*(5)	10.4	24.4	33.5	36.6
TRF*(10)	10.6	23.4	31.8	35.0
TRF*(15)	10.9	23.1	31.0	34.0
TRF*(20)	11.3	22.3	30.3	32.4
TRF*(25)	11.2	22.4	29.2	30.9
SWW (2.1)	10.7	18.3	51.3	48.6
SWW (2.4)	10.4	18.7	53.8	51.1
SWF*(1.8)	10.1	21.9	28.9	30.9
SWF*(2.1)	10.4	22.3	30.0	32.6
SWF*(2.4)	10.4	22.9	31.0	34.1
SBM	10.2	16.4	49.3	44.7

TABLE 25

Powers of some statistics for unequal variances under 10% 5N

Legend

TRW(100g) : Trimmed W, $W_t(g)$
TRF*(100g): Trimmed F*, $F_t^*(g)$
SWW(k) : Sine-wave W, $W_S(k)$
SWF*(k) : Sine-wave F*, $F_S^*(k)$
SBM : Studentized Brown and Mood

TABLE 25-(1)

DISTRIBUTION: 10% 5N
SAMPLE SIZES= 10 15 15 20
SIGMAS= 1.0 1.0 1.0 1.0
MEANS
H(1): 0.0 0.0 0.0 0.0
H(2): 1.0 0.0 0.0 0.0
H(3): 0.0 0.0 0.0 0.7
H(4): 0.5 0.0 0.0 0.5

ALPHA 1	H(1)	H(2)	H(3)	H(4)
TRW (0)	0.3	58.2	41.4	22.1
TRW (5)	0.4	64.9	57.5	29.5
TRW (10)	0.8	74.3	63.7	34.2
TRW (15)	0.8	72.7	62.8	34.2
TRF*(0)	0.5	45.2	33.2	16.3
TRF*(5)	0.4	64.1	51.7	26.6
TRF*(10)	0.7	77.0	64.3	34.8
TRF*(15)	0.7	79.5	65.9	37.0
TRF*(20)	0.7	77.5	63.9	35.3
TRF*(25)	0.9	73.1	59.6	32.1
SWW (2.1)	1.0	71.6	58.9	29.9
SWW (2.4)	0.9	74.6	61.7	33.1
SWF*(1.8)	0.6	76.9	63.3	32.7
SWF*(2.1)	0.6	78.0	64.4	34.2
SWF*(2.4)	0.7	79.0	65.7	36.1
SBM	0.7	36.0	52.1	26.3

TABLE 25-(1) (continued)

ALPHA	H(1)	H(2)	H(3)	H(4)
5				
TRW (0)	3.2	74.9	63.8	45.9
TRW (5)	3.6	81.6	79.8	56.1
TRW (10)	4.2	89.2	85.3	62.2
TRW (15)	4.5	89.3	86.1	62.9
TRF*(0)	3.3	67.3	55.9	35.9
TRF*(5)	3.4	82.9	75.6	50.2
TRF*(10)	4.1	91.6	84.6	60.7
TRF*(15)	4.5	92.7	87.2	63.1
TRF*(20)	4.6	92.7	86.5	61.7
TRF*(25)	5.0	91.4	84.8	59.7
SWW (2.1)	4.8	89.9	85.3	61.7
SWW (2.4)	4.6	90.3	86.1	63.3
SWF*(1.8)	4.1	92.4	86.6	61.4
SWF*(2.1)	3.9	92.5	87.6	62.3
SWF*(2.4)	3.9	92.6	87.5	62.9
SBM	3.9	72.2	75.9	51.1
10				
TRW (0)	7.5	81.1	73.3	59.1
TRW (5)	8.1	87.4	87.6	69.6
TRW (10)	9.1	93.9	92.2	75.5
TRW (15)	9.9	94.0	92.9	75.7
TRF*(0)	8.2	77.3	68.0	48.4
TRF*(5)	8.2	89.2	84.6	63.4
TRF*(10)	8.6	94.9	91.3	72.7
TRF*(15)	9.4	95.9	93.7	75.0
TRF*(20)	9.6	96.2	93.3	73.8
TRF*(25)	9.6	95.6	92.0	72.9
SWW (2.1)	9.7	94.8	93.1	75.2
SWW (2.4)	9.4	94.8	93.3	76.3
SWF*(1.8)	8.7	96.3	93.4	74.1
SWF*(2.1)	8.6	96.3	93.4	75.2
SWF*(2.4)	8.8	95.9	93.8	75.1
SBM	8.5	84.6	86.8	66.3

TABLE 25-(2)

DISTRIBUTION: 10% 5N
SAMPLE SIZES= 10 15 15 20
SIGMAS= 1.0 2.0 2.0 3.0
MEANS
H(1): 0.0 0.0 0.0 0.0
H(2): 1.0 0.0 0.0 0.0
H(3): 0.0 0.0 0.0 1.0
H(4): 1.0 0.0 0.0 1.0

ALPHA	H(1)	H(2)	H(3)	H(4)
1				
TRW (0)	0.5	32.7	8.6	31.2
TRW (5)	0.6	41.2	12.5	39.3
TRW (10)	0.8	49.7	14.6	47.1
TRW (15)	1.0	49.2	14.2	46.4
TRF*(0)	0.8	5.4	15.3	16.7
TRF*(5)	1.0	9.0	23.0	26.7
TRF*(10)	1.2	10.8	28.5	32.5
TRF*(15)	1.4	11.6	30.0	33.8
TRF*(20)	1.6	11.6	28.7	32.1
TRF*(25)	1.8	10.8	26.5	29.8
SWW (2.1)	0.9	46.8	11.5	43.0
SWW (2.4)	0.9	50.0	13.0	45.9
SWF*(1.8)	1.4	9.2	25.6	29.4
SWF*(2.1)	1.2	9.4	27.0	31.0
SWF*(2.4)	1.2	10.1	28.3	32.0
SBM	0.7	12.6	8.9	29.5
5				
TRW (0)	2.8	56.3	24.2	55.0
TRW (5)	3.5	66.3	32.0	65.1
TRW (10)	4.1	74.2	34.7	72.5
TRW (15)	4.4	75.4	34.6	73.1
TRF*(0)	4.1	19.4	31.0	35.0
TRF*(5)	4.4	28.4	42.8	49.2
TRF*(10)	5.1	33.9	48.4	56.3
TRF*(15)	5.6	35.4	49.8	58.2
TRF*(20)	5.9	34.3	48.5	57.2
TRF*(25)	6.2	32.7	46.9	55.5
SWW (2.1)	4.5	74.3	32.5	72.2
SWW (2.4)	4.6	75.7	34.4	73.6
SWF*(1.8)	4.8	31.5	47.5	55.7
SWF*(2.1)	4.9	32.9	48.8	56.6
SWF*(2.4)	4.9	34.2	49.7	57.2
SBM	4.2	40.9	25.3	55.7

TABLE 25-(2) (continued)

ALPHA 10	H(1)	H(2)	H(3)	H(4)
TRW (0)	7.1	67.8	35.4	67.0
TRW (5)	7.9	76.7	44.8	76.8
TRW (10)	8.8	83.2	48.0	82.6
TRW (15)	9.2	84.7	48.3	83.5
TRF*(0)	9.4	32.0	40.7	46.8
TRF*(5)	8.9	44.0	54.2	61.4
TRF*(10)	10.0	50.4	60.1	68.7
TRF*(15)	10.5	53.5	61.1	70.9
TRF*(20)	10.5	51.8	59.9	69.9
TRF*(25)	11.2	49.8	58.0	68.7
SWW (2.1)	8.9	84.6	47.3	83.8
SWW (2.4)	9.0	85.0	48.1	84.5
SWF*(1.8)	9.1	50.9	59.4	69.2
SWF*(2.1)	9.3	51.7	61.1	70.1
SWF*(2.4)	9.2	51.8	60.9	69.9
SBM	8.8	61.3	39.3	71.1

TABLE 25-(3)

DISTRIBUTION: 10% 5N
SAMPLE SIZES= 10 15 15 20
SIGMAS= 3.0 2.0 2.0 1.0
MEANS
H(1): 0.0 0.0 0.0 0.0
H(2): 1.0 0.0 0.0 0.0
H(3): 0.0 0.0 0.0 1.0
H(4): 1.0 0.0 0.0 1.0

ALPHA		H(1)	H(2)	H(3)	H(4)
1					
	TRW (0)	0.4	6.5	37.6	34.2
	TRW (5)	0.8	6.8	51.1	46.5
	TRW (10)	1.0	8.8	57.8	52.0
	TRW (15)	1.0	8.8	56.8	51.3
	TRF*(0)	0.5	7.9	11.1	14.5
	TRF*(5)	0.7	10.9	16.5	20.4
	TRF*(10)	1.0	14.2	22.2	25.9
	TRF*(15)	1.0	14.1	22.4	26.3
	TRF*(20)	1.2	14.2	21.4	25.3
	TRF*(25)	1.2	13.1	19.2	23.0
	SWW (2.1)	1.2	8.1	52.2	46.4
	SWW (2.4)	1.2	8.8	56.3	50.9
	SWF*(1.8)	1.0	11.7	17.9	22.4
	SWF*(2.1)	0.8	12.3	19.0	24.0
	SWF*(2.4)	0.9	13.6	21.3	26.2
	SBM	0.8	5.2	53.3	47.5
5					
	TRW (0)	2.9	18.1	63.2	59.7
	TRW (5)	3.4	18.9	76.8	72.3
	TRW (10)	4.6	22.0	83.0	78.7
	TRW (15)	4.9	22.4	83.2	79.3
	TRF*(0)	3.3	20.9	30.3	33.5
	TRF*(5)	3.9	26.1	40.1	43.2
	TRF*(10)	4.6	30.1	49.4	51.2
	TRF*(15)	5.0	30.3	50.4	52.5
	TRF*(20)	5.9	29.6	49.0	51.2
	TRF*(25)	5.7	28.3	46.8	48.7
	SWW (2.1)	4.5	21.4	81.8	77.9
	SWW (2.4)	4.8	22.3	83.2	79.3
	SWF*(1.8)	4.8	28.0	46.8	49.1
	SWF*(2.1)	4.8	28.9	48.5	50.6
	SWF*(2.4)	4.8	30.2	50.5	52.9
	SBM	4.8	18.2	77.3	72.3

TABLE 25-(3) (continued)

ALPHA	H(1)	H(2)	H(3)	H(4)
10				
TRW (0)	7.4	27.5	74.0	70.9
TRW (5)	7.6	29.2	85.9	82.6
TRW (10)	8.9	32.8	90.2	87.8
TRW (15)	9.4	33.0	91.2	88.4
TRF*(0)	7.7	31.4	44.1	46.8
TRF*(5)	8.5	36.4	55.6	57.1
TRF*(10)	9.1	40.2	65.3	65.4
TRF*(15)	9.7	41.3	66.5	66.9
TRF*(20)	10.4	39.9	64.7	65.3
TRF*(25)	10.5	39.1	62.3	63.2
SWW (2.1)	9.3	32.5	91.1	88.8
SWW (2.4)	9.0	33.5	91.4	89.2
SWF*(1.8)	9.3	39.1	63.9	64.9
SWF*(2.1)	9.2	39.8	65.3	66.4
SWF*(2.4)	9.6	40.5	66.9	67.8
SBM	9.8	28.2	87.2	83.1

TABLE 26

Powers of some statistics for unequal variances under 10% 10N

Legend

TRW(100g) : Trimmed W, $W_t(g)$
TRF*(100g): Trimmed F*, $F_t^*(g)$
SWW(k) : Sine-wave W, $W_s(k)$
SWF*(k) : Sine-wave F*, $F_s^*(k)$
SBM : Studentized Brown and Mood

TABLE 26-(1)

DISTRIBUTION: 10% 10N
SAMPLE SIZES= 10 15 15 20
SIGMAS= 1.0 1.0 1.0 1.0
MEANS
H(1): 0.0 0.0 0.0 0.0
H(2): 0.3 0.0 0.0 0.0
H(3): 0.0 0.0 0.0 0.3
H(4): 0.3 0.0 0.0 0.3

ALPHA	H(1)	H(2)	H(3)	H(4)
1				
TRW (0)	0.3	9.0	9.3	14.1
TRW (5)	0.3	11.8	20.4	22.5
TRW (10)	0.6	18.4	30.9	35.5
TRW (15)	0.7	19.1	31.3	37.3
TRF*(0)	0.2	1.8	5.2	6.4
TRF*(5)	0.2	5.6	12.0	15.0
TRF*(10)	0.5	14.0	27.1	31.3
TRF*(15)	0.7	17.7	32.1	37.6
TRF*(20)	0.7	17.6	32.3	37.6
TRF*(25)	0.9	16.8	30.0	34.9
SWW (2.1)	1.0	20.0	31.2	37.6
SWW (2.4)	1.0	21.6	34.6	41.5
SWF*(1.8)	0.7	18.4	34.7	40.1
SWF*(2.1)	0.7	19.5	36.8	42.6
SWF*(2.4)	0.8	20.6	38.5	44.3
SBM	0.6	7.0	23.5	28.6

TABLE 26-(1) (continued)

ALPHA	H(1)	H(2)	H(3)	H(4)
5				
TRW (0)	1.6	22.3	23.7	31.2
TRW (5)	2.2	27.4	43.5	47.9
TRW (10)	3.8	38.8	56.8	63.9
TRW (15)	4.0	39.9	59.3	67.2
TRF*(0)	2.3	9.2	14.8	18.1
TRF*(5)	2.1	18.4	28.8	32.6
TRF*(10)	3.0	31.7	48.9	54.9
TRF*(15)	3.7	38.4	57.4	62.8
TRF*(20)	4.2	40.1	59.9	65.0
TRF*(25)	4.6	38.7	57.7	63.7
SWW (2.1)	4.7	44.0	63.2	70.3
SWW (2.4)	4.7	45.6	64.4	71.9
SWF*(1.8)	4.1	42.2	64.4	70.4
SWF*(2.1)	4.1	43.4	65.1	70.8
SWF*(2.4)	4.1	44.4	65.8	71.6
SBM	3.1	24.7	48.1	53.4
10				
TRW (0)	4.9	33.0	35.6	43.5
TRW (5)	5.2	38.1	56.8	61.5
TRW (10)	7.5	51.2	69.7	76.1
TRW (15)	8.5	52.4	72.3	78.8
TRF*(0)	7.1	18.6	24.2	27.7
TRF*(5)	5.2	29.2	42.1	45.6
TRF*(10)	6.7	44.1	61.3	67.2
TRF*(15)	7.7	51.1	69.3	74.5
TRF*(20)	8.8	53.3	72.6	77.1
TRF*(25)	9.0	52.1	71.8	75.8
SWW (2.1)	9.4	57.2	76.7	82.7
SWW (2.4)	9.2	58.5	77.5	83.7
SWF*(1.8)	8.5	56.8	76.9	81.8
SWF*(2.1)	8.5	57.7	77.8	82.0
SWF*(2.4)	8.4	58.1	77.4	82.1
SBM	7.8	38.5	63.4	69.0

TABLE 26-(2)

DISTRIBUTION: 10% 1ON
SAMPLE SIZES= 10 15 15 20
SIGMAS= 1.0 2.0 2.0 3.0
MEANS
H(1): 0.0 0.0 0.0 0.0
H(2): 0.5 0.0 0.0 0.0
H(3): 0.0 0.0 0.0 0.5
H(4): 0.5 0.0 0.0 0.5

ALPHA		H(1)	H(2)	H(3)	H(4)
1					
	TRW (0)	0.2	13.4	3.5	11.6
	TRW (5)	0.4	19.8	6.5	18.0
	TRW (10)	0.6	32.5	9.1	29.2
	TRW (15)	0.9	34.7	9.7	32.1
	TRF*(0)	0.2	1.0	3.6	4.0
	TRF*(5)	0.3	3.2	8.7	10.3
	TRF*(10)	0.8	6.1	17.9	18.2
	TRF*(15)	0.9	7.7	21.6	22.2
	TRF*(20)	1.0	8.3	22.1	22.7
	TRF*(25)	1.1	7.9	20.6	21.8
	SWW (2.1)	0.8	36.9	9.1	32.1
	SWW (2.4)	0.7	39.6	10.6	35.3
	SWF*(1.8)	0.9	7.2	21.6	23.0
	SWF*(2.1)	1.0	7.8	23.2	24.5
	SWF*(2.4)	1.0	8.6	24.3	25.8
	SBM	0.6	9.4	6.8	18.9
5					
	TRW (0)	1.6	29.2	10.8	28.4
	TRW (5)	2.1	39.1	19.6	39.1
	TRW (10)	3.3	58.3	26.5	55.0
	TRW (15)	4.3	60.8	28.4	58.3
	TRF*(0)	2.4	7.0	12.8	13.0
	TRF*(5)	2.1	12.8	22.6	24.8
	TRF*(10)	3.3	20.4	34.6	37.6
	TRF*(15)	4.6	25.5	40.7	43.8
	TRF*(20)	5.2	26.3	41.2	44.7
	TRF*(25)	5.3	25.6	40.0	43.0
	SWW (2.1)	4.2	66.4	29.2	63.2
	SWW (2.4)	4.2	67.5	30.6	64.7
	SWF*(1.8)	4.1	25.7	42.9	46.3
	SWF*(2.1)	4.1	26.6	43.8	48.1
	SWF*(2.4)	4.0	28.0	44.0	48.8
	SBM	3.3	31.9	20.8	42.8

TABLE 26-(2) (continued)

ALPHA 10	H(1)	H(2)	H(3)	H(4)
TRW (0)	4.4	40.4	19.6	40.2
TRW (5)	5.0	53.0	30.9	52.1
TRW (10)	7.8	69.8	39.4	68.0
TRW (15)	9.1	73.0	41.1	71.8
TRF*(0)	7.0	14.3	21.0	21.7
TRF*(5)	5.4	22.8	33.2	35.8
TRF*(10)	7.1	33.4	45.2	50.1
TRF*(15)	8.9	39.9	51.1	56.2
TRF*(20)	9.9	40.7	52.4	57.7
TRF*(25)	10.1	40.0	51.3	56.2
SWW (2.1)	8.4	78.8	42.9	76.1
SWW (2.4)	8.6	79.0	44.0	77.1
SWF*(1.8)	8.2	42.7	54.3	60.0
SWF*(2.1)	8.2	44.3	55.3	61.1
SWF*(2.4)	8.5	44.7	55.6	61.6
SBM	7.5	48.6	32.8	58.3

TABLE 26-(3)

DISTRIBUTION: 10% 10N
SAMPLE SIZES= 10 15 15 20
SIGMAS= 3.0 2.0 2.0 1.0
MEANS
H(1): 0.0 0.0 0.0 0.0
H(2): 0.5 0.0 0.0 0.0
H(3): 0.0 0.0 0.0 0.5
H(4): 0.5 0.0 0.0 0.5

ALPHA		H(1)	H(2)	H(3)	H(4)
1					
	TRW (0)	0.3	3.0	11.6	10.8
	TRW (5)	0.4	3.7	24.0	20.6
	TRW (10)	0.9	5.7	38.1	33.4
	TRW (15)	0.9	5.8	40.2	35.6
	TRF*(0)	0.3	1.6	2.9	3.5
	TRF*(5)	0.3	3.5	4.9	6.5
	TRF*(10)	1.0	7.8	11.1	13.8
	TRF*(15)	1.2	9.3	12.8	15.9
	TRF*(20)	1.5	10.0	14.1	16.0
	TRF*(25)	1.5	9.2	13.1	14.9
	SWW (2.1)	1.3	5.9	40.2	35.2
	SWW (2.4)	1.1	6.4	44.9	39.2
	SWF*(1.8)	1.3	9.2	12.8	14.8
	SWF*(2.1)	1.3	9.6	14.0	16.6
	SWF*(2.4)	1.3	10.4	15.3	18.7
	SBM	0.9	3.6	37.3	33.1
5					
	TRW (0)	2.1	10.4	29.3	28.2
	TRW (5)	2.5	10.8	48.9	45.1
	TRW (10)	4.1	15.4	66.4	61.1
	TRW (15)	4.7	16.7	70.0	65.6
	TRF*(0)	2.3	7.4	11.1	12.1
	TRF*(5)	2.2	12.3	16.8	18.9
	TRF*(10)	3.5	19.4	30.0	34.2
	TRF*(15)	4.6	22.4	34.8	38.2
	TRF*(20)	5.3	23.3	36.1	39.3
	TRF*(25)	5.6	22.7	34.7	37.4
	SWW (2.1)	5.0	17.7	73.5	68.6
	SWW (2.4)	4.9	18.5	75.4	70.1
	SWF*(1.8)	5.1	23.1	36.6	40.3
	SWF*(2.1)	5.0	23.8	38.4	42.1
	SWF*(2.4)	5.2	24.9	40.2	43.5
	SBM	4.4	13.5	64.3	58.1

TABLE 26-(3) (continued)

ALPHA 10	H(1)	H(2)	H(3)	H(4)
TRW (0)	5.5	17.7	41.3	40.9
TRW (5)	5.7	18.8	61.9	58.4
TRW (10)	7.7	25.2	78.0	73.6
TRW (15)	9.1	25.9	82.4	77.4
TRF*(0)	6.5	15.0	19.7	21.1
TRF*(5)	5.8	20.3	28.7	30.2
TRF*(10)	7.3	28.8	44.6	47.3
TRF*(15)	8.6	31.7	50.2	52.3
TRF*(20)	10.0	33.0	52.3	54.0
TRF*(25)	9.9	32.9	51.2	52.7
SWW (2.1)	10.0	27.3	85.7	81.8
SWW (2.4)	9.9	28.3	86.5	82.7
SWF*(1.8)	9.6	33.7	53.6	57.0
SWF*(2.1)	9.4	34.3	55.2	58.0
SWF*(2.4)	9.4	35.3	56.8	59.5
SBM	9.3	21.6	76.5	71.5

TABLE 27

Powers of some statistics for unequal variances under 25% 1/U

Legend

TRW(100g)	:	Trimmed W, $W_t(g)$
TRF*(100g)	:	Trimmed F*, $F_t^*(g)$
SWW(k)	:	Sine-wave W, $W_s(k)$
SWF*(k)	:	Sine-wave F*, $F_s^*(k)$
SBM	:	Studentized Brown and Mood

TABLE 27-(1)

DISTRIBUTION: 25% 1/U

SAMPLE SIZES= 10 15 15 20

SIGMAS= 1.0 1.0 1.0 1.0

MEANS

H(1):	0.0	0.0	0.0	0.0
H(2):	1.0	0.0	0.0	0.0
H(3):	0.0	0.0	0.0	0.7
H(4):	0.5	0.0	0.0	0.5

ALPHA 1	H(1)	H(2)	H(3)	H(4)
TRW (0)	0.4	17.9	8.1	3.4
TRW (5)	0.5	21.0	13.2	5.4
TRW (10)	0.9	27.9	17.3	7.9
TRW (15)	1.0	28.1	17.7	8.0
TRF*(0)	0.3	6.6	5.2	2.0
TRF*(5)	0.5	13.2	10.3	4.3
TRF*(10)	0.8	24.1	18.5	7.9
TRF*(15)	0.8	26.4	19.9	8.4
TRF*(20)	1.0	26.3	19.3	8.6
TRF*(25)	0.8	24.4	17.8	8.3
SWW (2.1)	1.0	26.1	14.8	7.1
SWW (2.4)	1.1	28.7	17.2	7.8
SWF*(1.8)	0.7	24.5	17.2	7.5
SWF*(2.1)	0.7	25.8	18.7	8.3
SWF*(2.4)	0.8	28.5	20.2	8.4
SBM	0.6	10.2	12.8	6.1

TABLE 27-(1) (continued)

ALPHA	H(1)	H(2)	H(3)	H(4)
5				
TRW (0)	2.6	36.5	22.2	14.0
TRW (5)	3.0	41.5	34.5	18.6
TRW (10)	4.0	52.3	42.0	24.1
TRW (15)	4.4	52.0	42.0	25.4
TRF*(0)	1.9	16.9	13.4	8.0
TRF*(5)	2.5	30.7	24.3	14.1
TRF*(10)	3.7	49.1	39.7	23.3
TRF*(15)	4.2	53.5	42.8	25.8
TRF*(20)	4.7	53.0	42.7	26.1
TRF*(25)	5.0	51.2	41.4	25.0
SWW (2.1)	4.1	53.4	40.7	24.4
SWW (2.4)	4.2	55.2	42.1	25.4
SWF*(1.8)	3.9	52.9	42.2	24.1
SWF*(2.1)	3.8	53.5	43.6	25.3
SWF*(2.4)	4.3	54.8	43.6	26.1
SBM	4.0	33.0	32.7	19.1
10				
TRW (0)	6.4	47.0	32.4	23.2
TRW (5)	6.7	52.6	47.8	29.8
TRW (10)	8.7	64.4	56.5	37.0
TRW (15)	9.1	65.3	56.9	37.9
TRF*(0)	4.9	25.0	21.4	14.3
TRF*(5)	6.0	41.7	34.8	23.2
TRF*(10)	8.3	62.3	52.3	35.5
TRF*(15)	9.1	66.8	56.5	38.0
TRF*(20)	9.9	66.6	57.0	38.1
TRF*(25)	9.8	65.9	55.5	37.3
SWW (2.1)	8.8	67.3	56.4	37.8
SWW (2.4)	9.0	68.4	57.5	38.6
SWF*(1.8)	8.4	67.1	57.2	37.5
SWF*(2.1)	8.5	68.1	57.9	38.5
SWF*(2.4)	8.9	68.3	58.1	38.8
SBM	8.2	48.9	46.6	31.5

TABLE 27-(2)

DISTRIBUTION: 25% 1/U
SAMPLE SIZES= 10 15 15 20
SIGMAS= 1.0 2.0 2.0 3.0
MEANS
H(1): 0.0 0.0 0.0 0.0
H(2): 1.0 0.0 0.0 0.0
H(3): 0.0 0.0 0.0 1.0
H(4): 1.0 0.0 0.0 1.0

ALPHA		H(1)	H(2)	H(3)	H(4)
1					
	TRW (0)	0.3	6.9	1.8	6.2
	TRW (5)	0.5	9.3	3.0	8.3
	TRW (10)	0.7	13.3	4.0	12.1
	TRW (15)	0.8	13.7	4.2	12.5
	TRF*(0)	0.3	1.1	2.9	2.4
	TRF*(5)	0.6	1.9	5.3	4.6
	TRF*(10)	1.1	3.2	8.8	8.3
	TRF*(15)	1.3	3.7	9.4	9.2
	TRF*(20)	1.3	4.0	9.5	9.0
	TRF*(25)	1.5	3.8	8.6	8.7
	SWW (2.1)	0.7	12.3	3.4	10.9
	SWW (2.4)	0.7	13.6	3.8	12.1
	SWF*(1.8)	1.0	2.7	7.7	7.8
	SWF*(2.1)	0.9	2.9	8.4	8.2
	SWF*(2.4)	1.0	3.1	8.9	9.1
	SBM	0.5	4.4	3.2	6.7
5					
	TRW (0)	2.7	20.2	7.7	18.1
	TRW (5)	3.0	24.5	11.0	22.4
	TRW (10)	4.2	31.9	13.5	30.2
	TRW (15)	4.8	33.2	14.2	31.0
	TRF*(0)	2.4	5.0	8.4	7.9
	TRF*(5)	3.0	8.0	13.7	13.9
	TRF*(10)	4.4	11.8	20.6	21.7
	TRF*(15)	5.1	13.0	22.6	23.7
	TRF*(20)	5.1	13.2	22.3	23.1
	TRF*(25)	5.6	13.2	21.5	22.4
	SWW (2.1)	4.5	32.9	13.0	30.6
	SWW (2.4)	4.6	34.2	13.6	31.7
	SWF*(1.8)	4.5	11.6	21.0	21.5
	SWF*(2.1)	4.7	12.0	21.3	22.1
	SWF*(2.4)	4.9	12.6	22.0	22.9
	SBM	3.5	16.7	10.7	19.8

TABLE 27-(2) (continued)

ALPHA	H(1)	H(2)	H(3)	H(4)
10				
TRW (0)	6.4	30.3	14.4	28.5
TRW (5)	7.6	36.0	19.8	33.6
TRW (10)	9.1	45.7	23.4	42.9
TRW (15)	9.4	46.7	23.4	44.0
TRF*(0)	5.3	9.7	13.2	13.6
TRF*(5)	6.8	15.0	20.8	22.2
TRF*(10)	9.1	21.5	29.7	32.3
TRF*(15)	10.2	23.2	31.8	34.2
TRF*(20)	10.2	23.0	31.9	34.5
TRF*(25)	10.1	22.6	31.6	33.9
SWW (2.1)	9.2	46.9	22.3	45.0
SWW (2.4)	9.6	47.8	23.2	46.4
SWF*(1.8)	8.6	20.8	30.5	32.8
SWF*(2.1)	9.2	21.8	31.0	33.2
SWF*(2.4)	9.3	22.1	31.6	33.9
SBM	8.2	28.7	18.6	33.2

TABLE 27-(3)

DISTRIBUTION: 25% 1/U
SAMPLE SIZES= 10 15 15 20
SIGMAS= 3.0 2.0 2.0 1.0
MEANS
H(1): 0.0 0.0 0.0 0.0
H(2): 1.0 0.0 0.0 0.0
H(3): 0.0 0.0 0.0 1.0
H(4): 1.0 0.0 0.0 1.0

ALPHA		H(1)	H(2)	H(3)	H(4)
1					
	TRW (0)	0.4	1.5	6.5	6.2
	TRW (5)	0.5	1.7	10.3	9.6
	TRW (10)	1.0	2.8	14.3	13.4
	TRW (15)	1.2	3.0	14.7	13.5
	TRF*(0)	0.3	1.3	1.4	2.1
	TRF*(5)	0.5	2.3	2.3	3.5
	TRF*(10)	1.0	4.1	4.3	5.9
	TRF*(15)	1.1	4.4	4.8	6.7
	TRF*(20)	1.3	4.7	5.1	7.3
	TRF*(25)	1.3	4.5	5.0	6.9
	SWW (2.1)	1.4	3.3	12.1	11.4
	SWW (2.4)	1.4	3.4	14.0	12.9
	SWF*(1.8)	1.1	3.8	4.0	5.8
	SWF*(2.1)	1.1	3.9	4.0	6.1
	SWF*(2.4)	1.2	4.5	4.4	6.6
	SBM	0.9	1.7	13.8	11.9
5					
	TRW (0)	2.6	6.3	19.5	19.1
	TRW (5)	3.1	7.3	29.0	26.9
	TRW (10)	4.5	9.6	36.9	34.0
	TRW (15)	5.0	9.7	38.3	35.0
	TRF*(0)	1.9	4.7	6.4	8.0
	TRF*(5)	2.8	7.4	10.0	12.0
	TRF*(10)	4.5	12.3	16.5	18.6
	TRF*(15)	4.9	13.6	17.9	20.2
	TRF*(20)	5.5	13.7	18.7	20.5
	TRF*(25)	5.7	13.5	18.0	20.6
	SWW (2.1)	5.0	9.8	36.7	34.2
	SWW (2.4)	4.9	10.2	38.6	35.9
	SWF*(1.8)	4.8	12.1	15.7	18.7
	SWF*(2.1)	4.8	12.6	16.4	19.9
	SWF*(2.4)	5.1	13.1	17.2	20.6
	SBM	4.7	8.3	33.4	30.5

TABLE 27-(3) (continued)

ALPHA	H(1)	H(2)	H(3)	H(4)
10				
TRW (0)	6.2	12.2	30.0	29.5
TRW (5)	6.6	13.4	42.8	39.6
TRW (10)	9.2	16.4	52.5	47.5
TRW (15)	9.7	18.2	54.0	48.9
TRF*(0)	5.2	9.9	12.4	14.2
TRF*(5)	6.1	13.5	18.9	20.4
TRF*(10)	8.9	19.4	27.9	29.6
TRF*(15)	9.8	21.2	29.3	31.7
TRF*(20)	10.7	21.9	29.8	32.1
TRF*(25)	11.1	21.1	29.4	31.3
SWW (2.1)	9.8	17.6	53.3	49.3
SWW (2.4)	9.9	18.0	54.3	50.8
SWF*(1.8)	10.1	20.2	28.0	30.7
SWF*(2.1)	10.0	20.7	29.4	31.6
SWF*(2.4)	10.4	21.6	30.0	32.9
SBM	10.1	15.5	48.2	44.1

TABLE 28

Powers of some statistics for unequal variances under CAUCHY

Legend

TRW(100g) : Trimmed W, $W_t(g)$
TRF*(100g): Trimmed F*, $F_t^*(g)$
SWW(k) : Sine-wave W, $W_s(k)$
SWF*(k) : Sine-wave F*, $F_s^*(k)$
SBM : Studentized Brown and Mood

TABLE 28-(1)

DISTRIBUTION: CAUCHY

SAMPLE SIZES=	10	15	15	20
SIGMAS=	1.0	1.0	1.0	1.0
MEANS				
H(1):	0.0	0.0	0.0	0.0
H(2):	1.0	0.0	0.0	0.0
H(3):	0.0	0.0	0.0	0.7
H(4):	0.5	0.0	0.0	0.5

ALPHA 1	H(1)	H(2)	H(3)	H(4)
TRW (0)	0.1	6.9	2.2	1.3
TRW (5)	0.1	11.6	5.1	2.0
TRW (10)	0.2	24.3	12.3	5.0
TRW (15)	0.2	28.9	17.5	7.2
TRF*(0)	0.1	1.2	0.8	0.5
TRF*(5)	0.2	3.5	2.4	1.2
TRF*(10)	0.2	12.4	9.0	3.5
TRF*(15)	0.4	19.8	14.4	6.0
TRF*(20)	0.6	28.2	19.2	8.6
TRF*(25)	0.6	30.3	21.0	9.3
SWW (2.1)	0.3	37.2	21.3	8.9
SWW (2.4)	0.3	37.6	21.9	9.2
SWF*(1.8)	0.5	28.1	19.6	8.4
SWF*(2.1)	0.5	27.3	18.9	8.6
SWF*(2.4)	0.5	27.6	18.8	8.9
SBM	0.5	11.4	23.2	12.0

TABLE 28-(1) (continued)

ALPHA	H(1)	H(2)	H(3)	H(4)
5				
TRW (0)	1.5	18.2	9.0	6.1
TRW (5)	1.3	25.0	17.7	9.7
TRW (10)	1.7	44.0	31.8	17.3
TRW (15)	2.1	50.2	41.1	23.0
TRF*(0)	1.3	5.4	4.8	3.5
TRF*(5)	1.4	11.9	9.1	5.4
TRF*(10)	2.4	29.9	23.5	12.4
TRF*(15)	2.7	42.7	33.7	18.4
TRF*(20)	3.1	53.3	42.2	24.1
TRF*(25)	3.1	57.2	45.8	26.0
SWW (2.1)	2.9	60.1	47.1	29.7
SWW (2.4)	3.1	59.4	46.5	29.3
SWF*(1.8)	3.2	54.3	43.7	25.1
SWF*(2.1)	3.4	52.9	42.4	24.6
SWF*(2.4)	3.8	52.1	41.4	24.4
SBM	3.7	34.5	47.7	30.3
10				
TRW (0)	4.2	27.0	16.1	12.2
TRW (5)	3.9	34.4	28.6	17.8
TRW (10)	5.4	54.7	44.4	29.9
TRW (15)	5.2	60.9	55.1	36.9
TRF*(0)	4.2	10.5	9.9	7.3
TRF*(5)	4.0	20.6	16.3	10.6
TRF*(10)	5.4	43.2	34.8	22.1
TRF*(15)	6.2	56.0	46.2	29.0
TRF*(20)	6.8	66.3	55.2	36.4
TRF*(25)	7.3	70.4	59.3	39.2
SWW (2.1)	7.2	70.2	62.4	44.1
SWW (2.4)	7.5	69.3	61.5	44.3
SWF*(1.8)	7.1	68.4	57.4	37.8
SWF*(2.1)	7.7	66.9	56.7	37.0
SWF*(2.4)	8.3	65.7	56.0	36.5
SBM	7.7	50.9	62.5	45.1

TABLE 28-(2)

DISTRIBUTION:CAUCHY
SAMPLE SIZES= 10 15 15 20
SIGMAS= 1.0 2.0 2.0 3.0
MEANS
H(1): 0.0 0.0 0.0 0.0
H(2): 1.0 0.0 0.0 0.0
H(3): 0.0 0.0 0.0 1.0
H(4): 1.0 0.0 0.0 1.0

ALPHA	H(1)	H(2)	H(3)	H(4)
1				
TRW (0)	0.1	2.3	0.6	2.1
TRW (5)	0.1	4.0	1.2	3.3
TRW (10)	0.1	9.2	2.5	7.4
TRW (15)	0.2	12.3	3.2	10.9
TRF*(0)	0.1	0.4	0.9	0.7
TRF*(5)	0.1	0.5	1.7	1.6
TRF*(10)	0.2	1.4	4.4	4.5
TRF*(15)	0.5	2.0	6.5	6.8
TRF*(20)	0.7	2.9	8.7	9.6
TRF*(25)	0.7	3.3	9.7	10.6
SWW (2.1)	0.3	16.4	3.6	15.9
SWW (2.4)	0.4	16.8	4.2	16.1
SWF*(1.8)	0.6	2.7	8.4	9.4
SWF*(2.1)	0.6	2.7	8.5	9.3
SWF*(2.4)	0.7	3.0	8.7	9.6
SBM	0.6	5.1	5.2	12.9
5				
TRW (0)	0.9	9.2	3.6	8.4
TRW (5)	1.1	12.8	5.7	11.8
TRW (10)	1.6	24.4	10.1	23.0
TRW (15)	2.0	30.7	12.9	29.9
TRF*(0)	1.6	2.8	3.7	3.2
TRF*(5)	1.6	3.4	6.1	6.2
TRF*(10)	2.5	7.2	13.2	13.6
TRF*(15)	2.9	10.0	18.4	19.2
TRF*(20)	3.4	12.8	22.2	25.1
TRF*(25)	3.5	15.0	24.3	27.9
SWW (2.1)	2.9	38.5	15.8	37.6
SWW (2.4)	3.0	38.5	15.8	37.1
SWF*(1.8)	3.6	12.7	22.4	24.8
SWF*(2.1)	3.7	12.3	21.9	24.7
SWF*(2.4)	4.0	12.3	21.8	24.9
SBM	3.4	19.6	17.1	31.6

TABLE 28-(2) (continued)

ALPHA	H(1)	H(2)	H(3)	H(4)
10				
TRW (0)	3.5	16.4	8.2	15.5
TRW (5)	3.3	21.8	11.9	20.5
TRW (10)	4.6	36.4	18.1	34.7
TRW (15)	5.2	43.9	22.2	42.9
TRF*(0)	5.1	6.1	7.1	7.4
TRF*(5)	4.9	7.5	10.9	11.8
TRF*(10)	6.0	13.9	20.8	22.0
TRF*(15)	6.7	18.3	28.3	29.7
TRF*(20)	7.3	22.1	33.2	36.2
TRF*(25)	7.7	25.1	35.6	40.2
SWW (2.1)	6.8	52.6	26.8	51.6
SWW (2.4)	6.9	51.8	26.7	51.2
SWF*(1.8)	7.5	22.4	33.4	36.2
SWF*(2.1)	8.0	22.3	32.7	35.5
SWF*(2.4)	8.5	22.0	31.7	34.7
SBM	7.7	33.4	28.1	47.0

TABLE 28-(3)

DISTRIBUTION: CAUCHY

SAMPLE SIZES=	10	15	15	20
SIGMAS=	3.0	2.0	2.0	1.0
MEANS				
H(1):	0.0	0.0	0.0	0.0
H(2):	1.0	0.0	0.0	0.0
H(3):	0.0	0.0	0.0	1.0
H(4):	1.0	0.0	0.0	1.0

ALPHA	H(1)	H(2)	H(3)	H(4)
1				
TRW (0)	0.1	0.7	1.6	1.7
TRW (5)	0.1	0.8	3.8	3.6
TRW (10)	0.2	1.8	9.0	7.9
TRW (15)	0.3	2.0	13.1	11.6
TRF*(0)	0.1	0.2	0.5	0.5
TRF*(5)	0.1	0.5	0.6	1.0
TRF*(10)	0.3	1.7	2.0	3.0
TRF*(15)	0.3	2.3	3.1	4.1
TRF*(20)	0.7	3.7	4.6	6.6
TRF*(25)	0.7	3.9	4.8	7.0
SWW (2.1)	0.7	3.7	17.2	14.4
SWW (2.4)	0.6	3.7	18.7	15.2
SWF*(1.8)	0.4	3.5	4.4	6.2
SWF*(2.1)	0.4	3.8	4.7	6.2
SWF*(2.4)	0.6	4.0	4.8	6.6
SBM	0.8	2.8	22.5	21.0
5				
TRW (0)	1.3	3.8	8.1	8.0
TRW (5)	1.4	4.0	14.5	13.8
TRW (10)	1.8	7.7	26.6	25.1
TRW (15)	2.1	8.6	35.1	33.0
TRF*(0)	1.3	2.1	2.8	3.2
TRF*(5)	1.4	3.4	4.0	4.8
TRF*(10)	2.0	7.3	9.4	11.4
TRF*(15)	2.3	9.7	12.0	14.6
TRF*(20)	3.4	12.3	16.9	20.0
TRF*(25)	3.3	13.1	17.7	21.3
SWW (2.1)	3.4	12.0	43.8	41.1
SWW (2.4)	3.5	12.3	43.4	41.0
SWF*(1.8)	3.1	13.0	17.2	19.9
SWF*(2.1)	3.3	12.9	16.9	19.4
SWF*(2.4)	3.7	13.2	17.7	19.9
SBM	4.5	10.8	46.9	44.2

TABLE 28-(3) (continued)

ALPHA 10	H(1)	H(2)	H(3)	H(4)
TRW (0)	3.8	8.2	15.0	14.4
TRW (5)	3.7	9.0	24.1	23.3
TRW (10)	5.1	14.4	40.2	38.4
TRW (15)	5.2	15.4	50.5	48.1
TRF*(0)	4.0	5.6	6.5	7.7
TRF*(5)	3.4	7.8	8.5	10.0
TRF*(10)	5.6	14.0	17.2	20.2
TRF*(15)	6.0	16.9	22.3	25.3
TRF*(20)	7.2	20.5	28.7	31.2
TRF*(25)	7.2	21.2	30.0	32.9
SWW (2.1)	7.6	20.4	60.4	57.2
SWW (2.4)	7.9	20.6	59.1	56.8
SWF*(1.8)	7.2	21.9	29.9	31.6
SWF*(2.1)	7.9	21.4	29.8	31.8
SWF*(2.4)	8.4	21.4	29.8	32.1
SBM	9.7	19.1	62.4	59.8

Chapter V

DISCUSSION AND FUTURE RESEARCH

5.1 DISCUSSION

In this dissertation we proposed multi-sample location procedures to test the equality of several group means and assessed their performance under long-tailed symmetric distributions ranging from normal to Cauchy. The trimmed F and the sine-wave F are proposed for equal variances, and the trimmed W, F* and the sine-wave W, F* for unequal variances. As shown in Chapter 4, these statistics are robust against symmetric long-tailedness.

Since these new statistics can easily be programmed, they can very well be incorporated into the general-purpose software statistical packages such as BMDP and SAS as an option to the users. If one statistic is to be chosen from each family, we would recommend the trimmed statistics with $g = 0.15$ and the sine-wave statistics with $k = 2.1$. These statistics perform reasonably well over a wide range of underlying distributions, both in the cases of equal and unequal variances. Of course, if one has prior knowledge of the tail-length of the parent distribution, one can choose the appropriate g or k best for that situation and come out better (obtain higher power) than the ones we recommend. In general, one should trim more or use a smaller k value as the tail gets longer. When variances are unequal, the choice between the W-type and F*-type statistics is according to whether extreme means are associated with large variances or not.

5.2 SUGGESTIONS FOR FUTURE RESEARCH

We have solved to a certain degree the problem of conservatism and inefficiency of the standard parametric statistics F, W and F* under long-tailed distributions, but many related problems remain unsolved. Some of them are given below:

1. The proposed procedures are derived assuming symmetry of the underlying distribution. One may then ask how well they perform under asymmetric distributions. Some work has been done on the performance of the two-sample trimmed t under skewed distributions [7, 31]. We imagine the performance of our proposed statistics will not be greatly affected by moderate skewness, but severe skewness will affect it considerably. For skewed distributions one may apply asymmetric trimming with different trimming proportion from each end of the ordered sample. Very little work has been done in this direction.

2. It is important to choose the right trimming proportion (g) when the trimmed statistic is used because the loss in power could be great if a wrong value of g is selected. One may try to estimate the tail-length from the sample and choose g accordingly. However, this will make the test procedure invalid because the trimmed statistic is based on the assumption that g is chosen beforehand. In order to use this method with the intended level of significance, some modifications (perhaps in the critical value or the statistic itself) have to be made. A two-stage sampling technique may be regarded as a possible solution for the problem of selecting the trimming proportion. In other words, we

estimate the tail-length in the first stage and using the second-stage samples which are independent of the first stage, we compute the trimmed statistic with g chosen according to the tail-length estimated in the first stage. Since we have an independent tail-length estimate, trimmed statistics can be used without any modification. However, this method is very impractical especially since estimates of tail-length are poor for small n which we are particularly concerned about. It also wastes information contained in the first-stage samples and hence, lowers the efficiency.

3. Using similar methods, we can also modify F, W and F* using other M-estimators such as Hampel's 3 point M-estimator [1] and Tukey's biweight M-estimator [19]. We expect that their performance will not be much different from that of the sine-wave statistic because of their similar performance in the case of one-sample interval estimation [10].

4. In order to obtain a reasonable approximation by an F-distribution to the trimmed and the sine-wave statistics, it is required that the sample size in each group be not too small ($n_i \geq 10$). It is possible to obtain good critical points for smaller n's by using Monte Carlo techniques even though it will require a tremendous amount of computation.

5. We have seen that the trimmed W with high g (≥ 0.2) and the sine-wave W with $k = 1.8$ are somewhat liberal in certain cases under normality. Another proposal for further research is to correct the liberalism of these statistics so that we can use them under extremely long-tailed situations.

6. Very often, upon rejecting the equality of group means, it is of interest to perform multiple comparisons of linear combinations of group means with an overall confidence. Similar to Scheffe's method, we may construct confidence intervals for $\sum_i a_i \mu_i$ as follows:

$$\sum_{i=1}^{c} a_i \hat{\mu}_i \pm S,$$

where $S^2 = f_1 \text{MSE}\, F(f_1, f_2; 1-\alpha) \sum_i a_i^2/l_i$ and $f_1 = c$. If the trimmed mean is used for $\hat{\mu}_i$, $\text{MSE} = \sum_i \text{SSD}_{iwg}/(H - c)$, $f_2 = H - c$ and $l_i = h_i$. If $\hat{\mu}_i$ is the sine-wave M-estimator, $\text{MSE} = \sum_i (n_i - 1)S_{ik}^2/(N - c)$, $f_2 = d_k(N - c)$ and l_i = some function of n_i ($d_k n_i$ may be tried). If we are interested in contrasts only, then the same formula may be used with $f_1 = c - 1$. We conjecture that the overall confidence of these intervals will be approximately $100(1 - \alpha)\%$. The verification of our conjecture and performance evaluation of the suggested multiple comparison procedures constitute another area of future work.

7. It was shown that the W-type statistic is more powerful than the F*-type when extreme means have small variances and vice versa for the opposite situation. One more suggestion for research is to develop a statistic between the W-type and the F*-type statistics which is reasonably powerful no matter which means have small variances.

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